Informational and topological signatures of individuality and age

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The Sense 23/11/2023

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at Northeastern University

PROJECT CETI

CENTAI

Higher-order signatures of individuality and age

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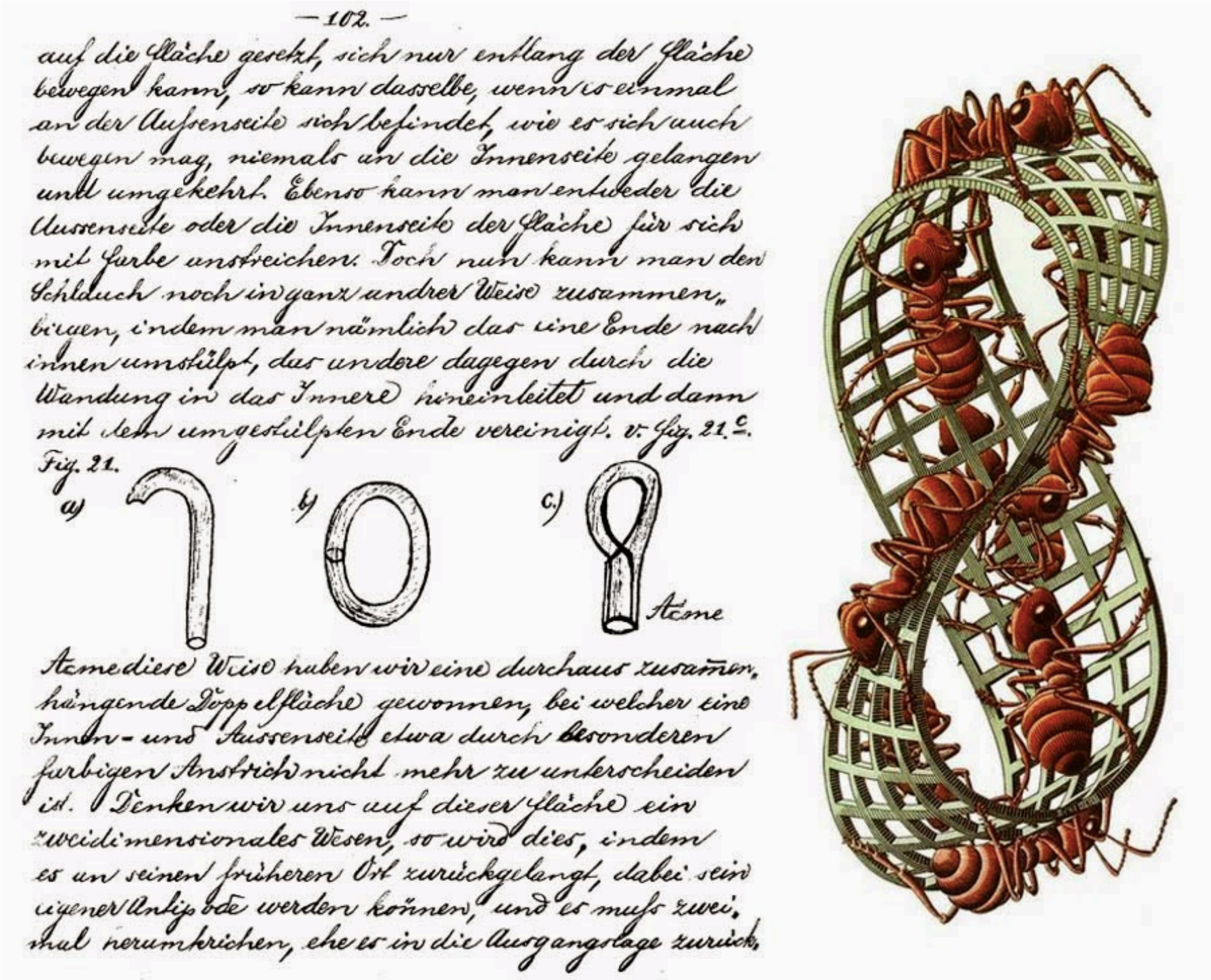
What is topology?


— 102. —

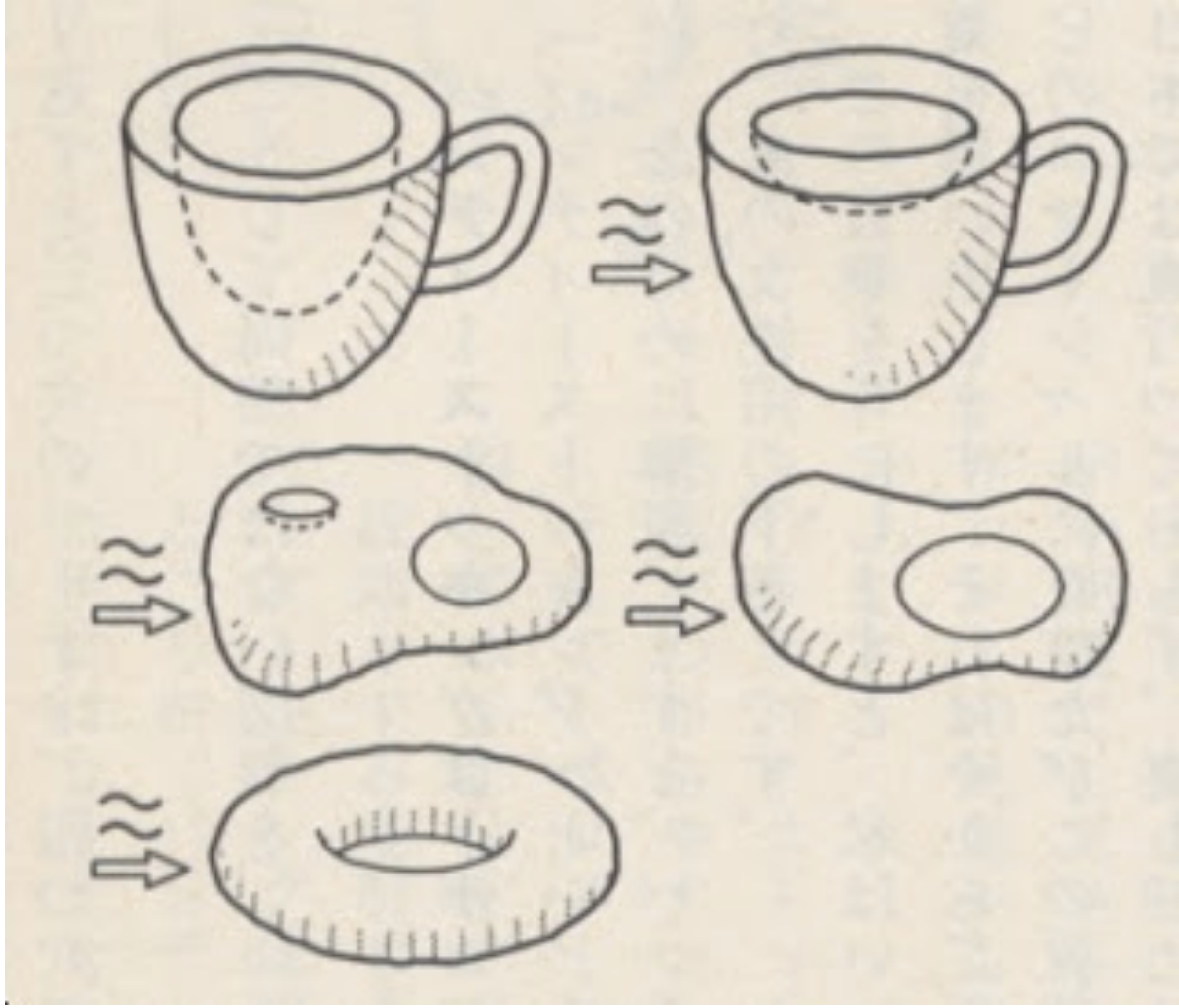
auf die Fläche gesetzt, sich nur entlang der Fläche
bewegen kann, so kann dasselbe, wenn es einmal
an der Aussenseite sich befindet, wie es sich auch
bewegen mag, niemals an die Innenseite gelangen
und umgekehrt. Ebenso kann man entweder die
Aussenseite oder die Innenseite der Fläche für sich
mit Farbe anstreichen. Doch nun kann man den
Schlauch noch in ganz andrer Weise zusammen-
biegen, indem man nämlich das eine Ende nach
innen umstülpt, das andere dagegen durch die
Wandung in das Innere hineinleitet und dann
mit dem umgestülpten Ende vereinigt. v. Fig. 21.c.

Fig. 21.

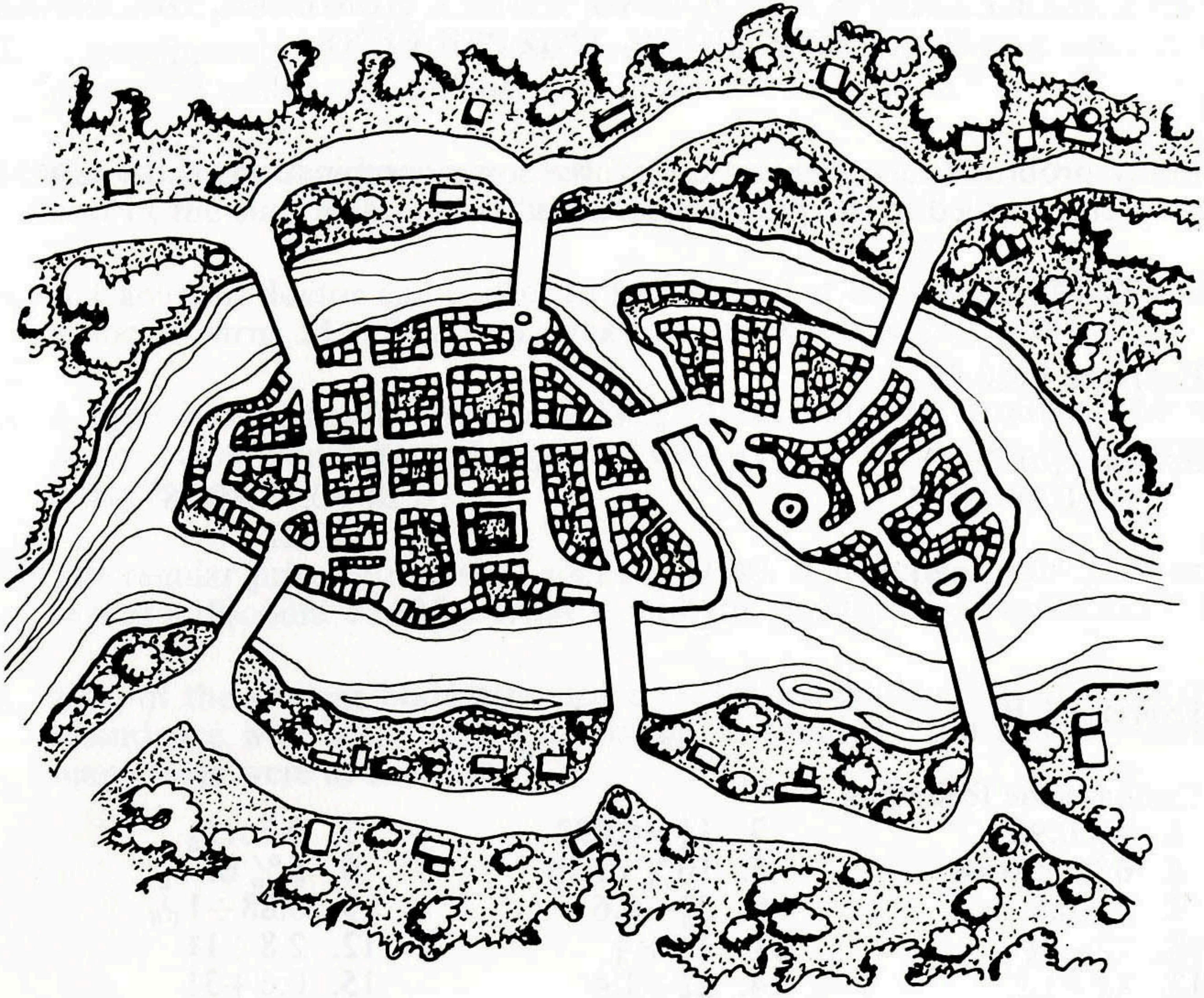
a) b) c)

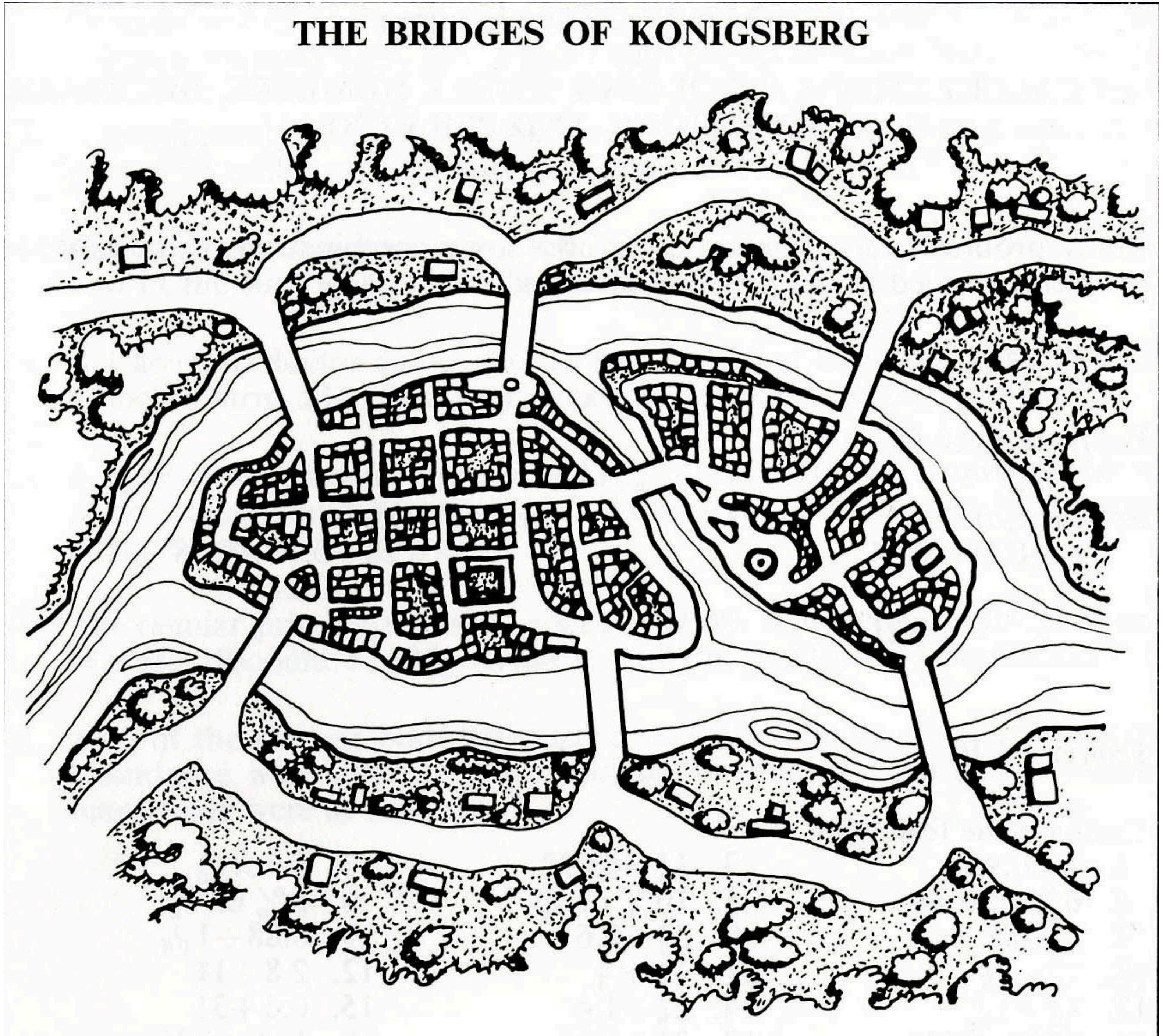
Acme

Acme diese Weise haben wir eine durchaus zusammen-
hängende Doppelfläche gewonnen, bei welcher eine
Innen- und Aussenseite etwa durch besonderen
farbigen Anstrich nicht mehr zu unterscheiden
ist. Denken wir uns auf dieser Fläche ein
zweidimensionales Wesen, so wird dies, indem
es an seinen früheren Ort zurückgelangt, dabei sein
eigener Antipode werden können, und es muss zwei-
mal herumkriechen, ehe es in die Ausgangslage zurück.



THE BRIDGES OF KONIGSBERG

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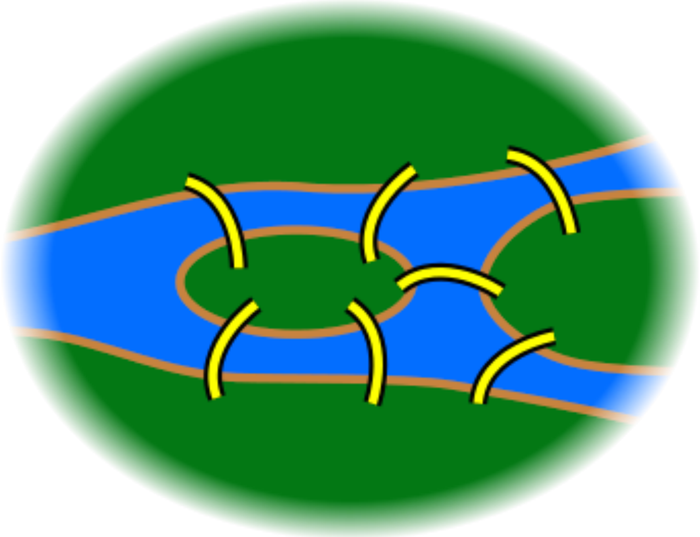
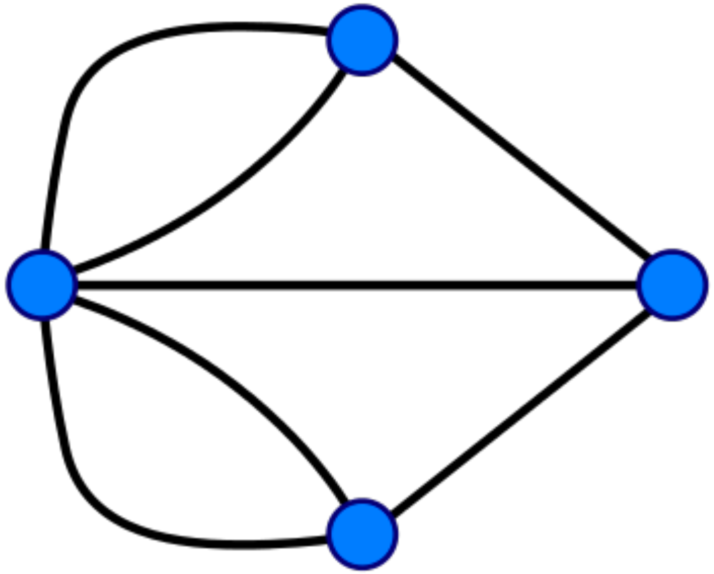


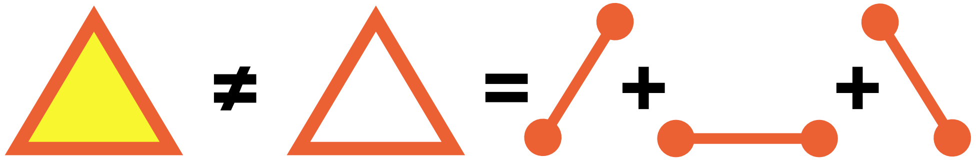


Why topology?

DOT
= 0-simplex

EDGE =
1-simplex

TRIANGLE
= 2-simplex



Definition of k-simplex

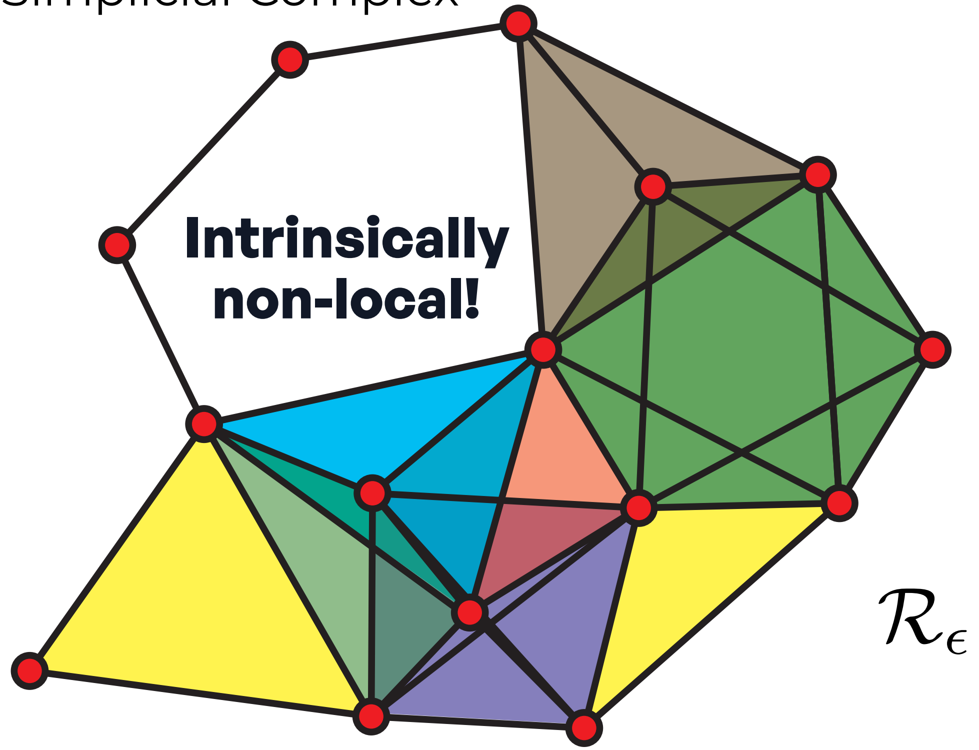
$$\sigma = [p_0, p_1, p_2, \ldots, p_{k-1}]$$

Multivariate information

$$P(\mathbf{X}) = P(X_0, X_1, X_2, \ldots, X_{k-1})$$

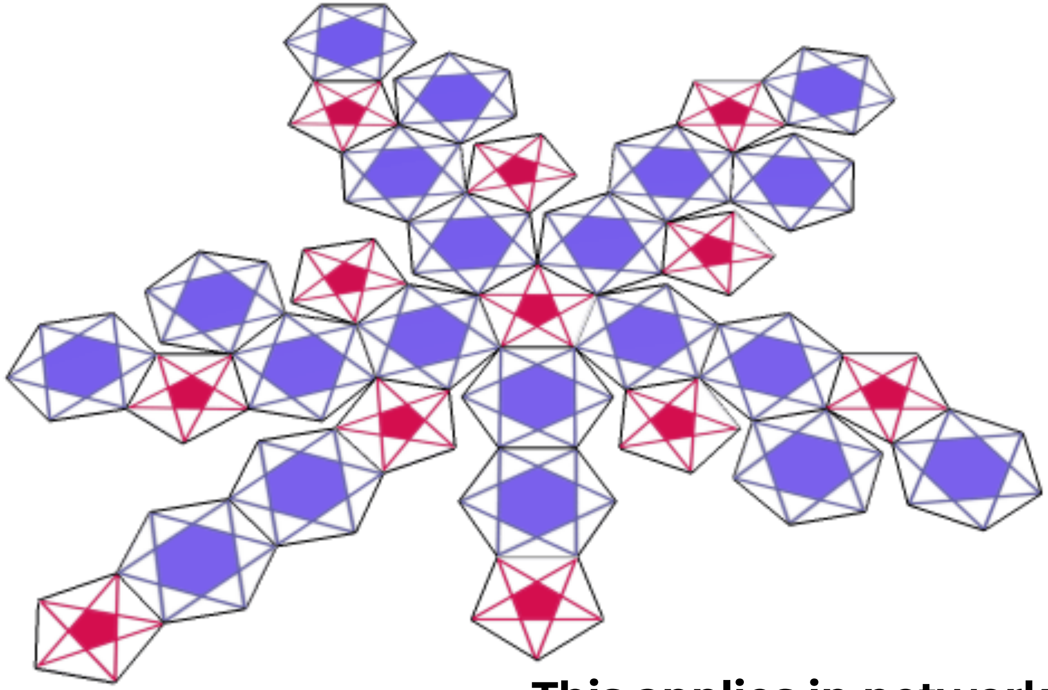
Intrinsically
higher-order!

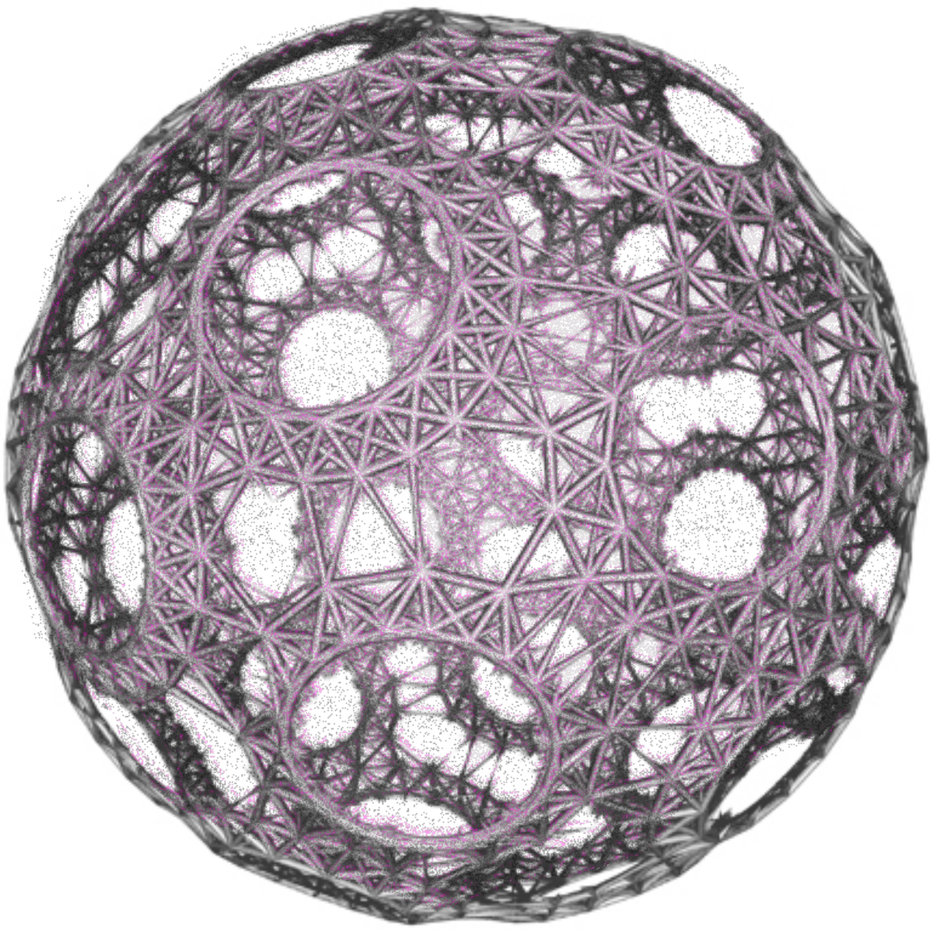
Simplicial Complex



Ghrist, R. (2008). Barcodes: The persistent topology of data. *Bulletin-American Mathematical Society*, *45*(1), 61.

Topology in the wild





This applies in networks as well as to data spaces

What does it mean in practice?

Persistent homology pipeline (Ghrist 2008)

Data of sorts

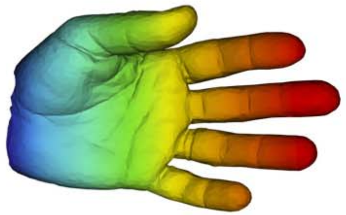
Filtration over distance/density/weights

Homological properties

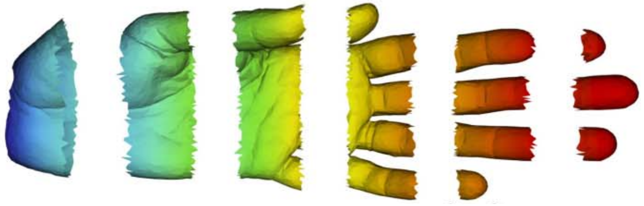
Mapper Pipeline (Singh et al 2007)

Point cloud

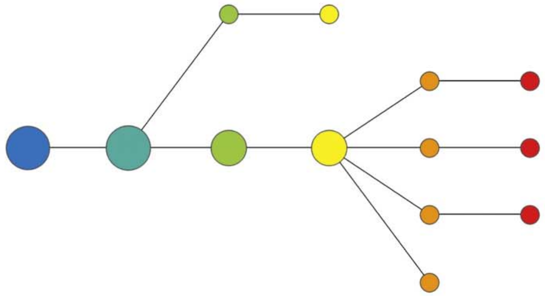
colouring (projecting)
using geometric filters



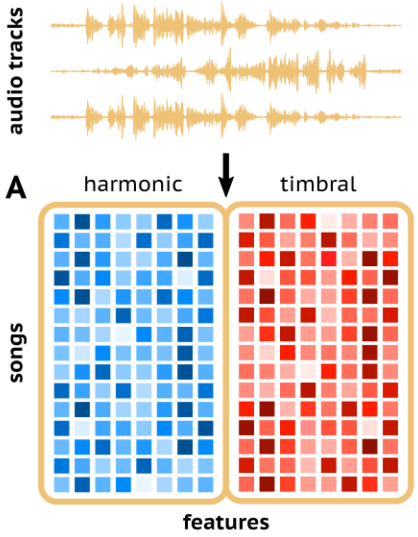
overlapped
binning

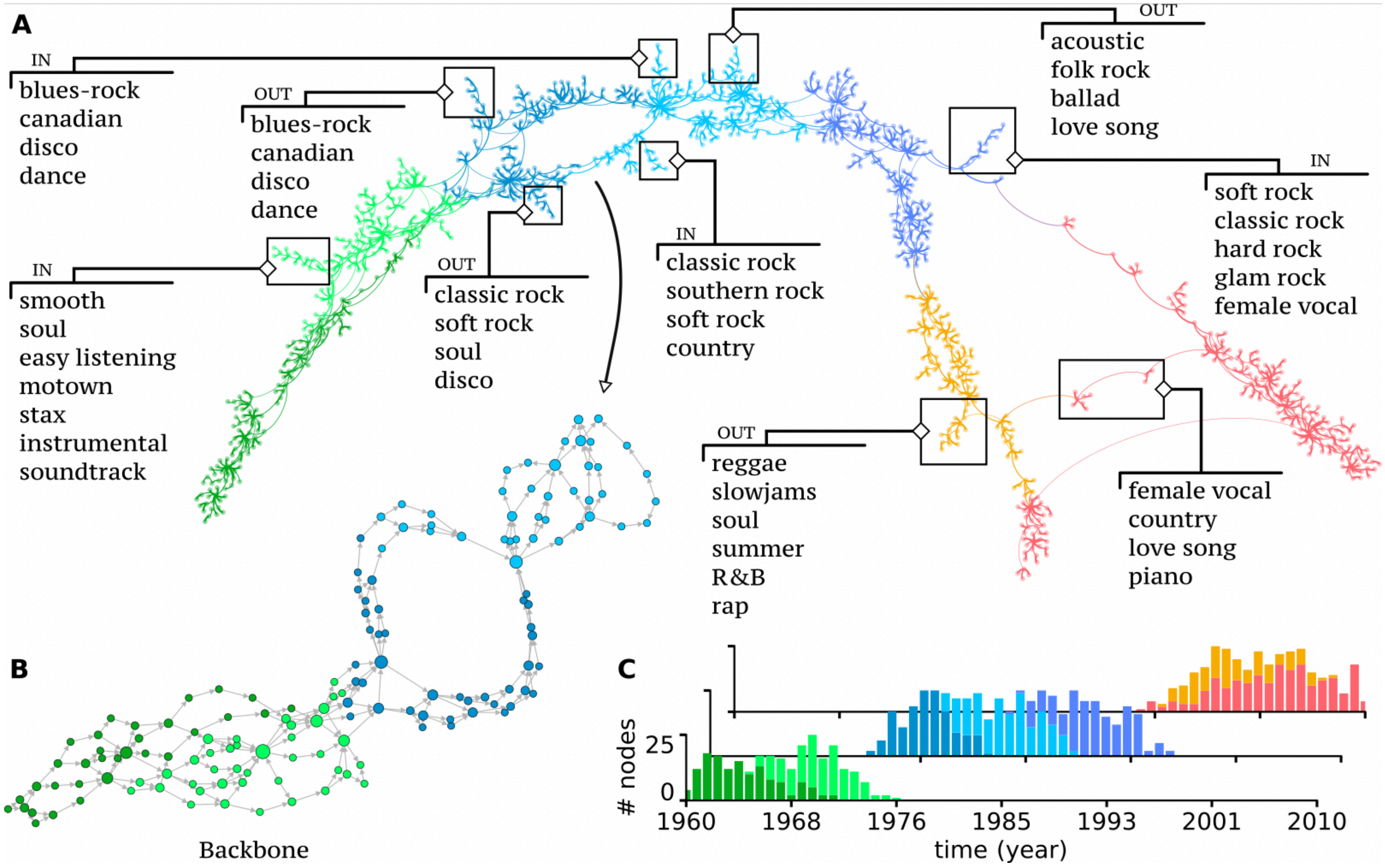


Clustering and
network construction binning



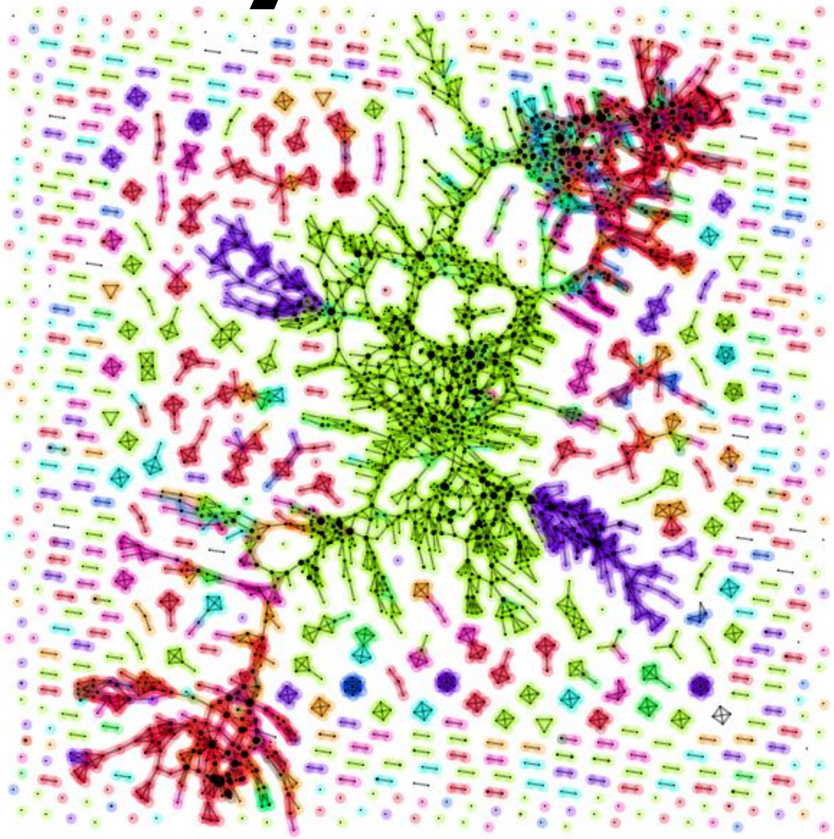
What does it mean in practice?



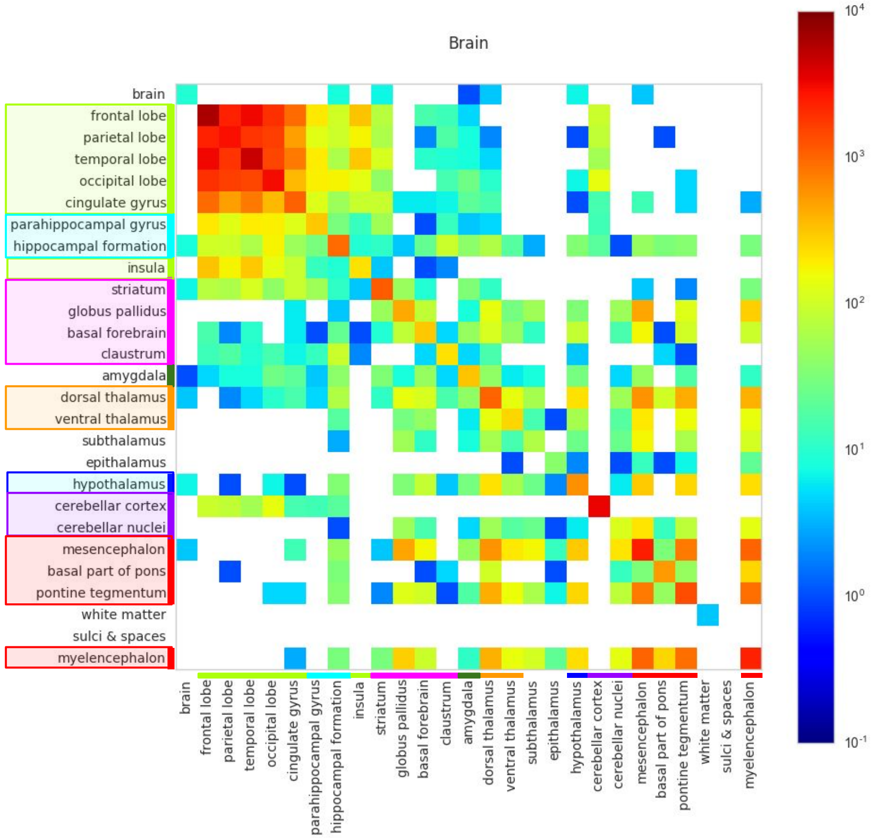


Iacopini, Petri, Latora, (2020) in eternal preparation

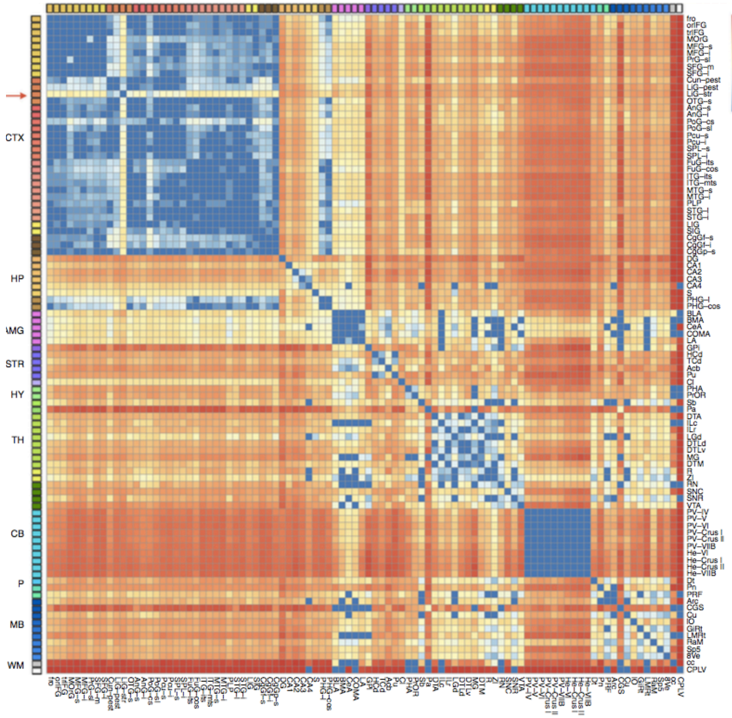
Do topological gene-backbones carry information?



basal_ganglia · hypothalamus · amygdala · thalamus
cerebellum · hippocampus · neocortex · brainstem



Hawrylycz, Michael, et al. "Canonical genetic signatures of the adult human brain." *Nature neuroscience* 18.12 (2015): 1832-1844.



Patania, Selvaggi, di Pasquale, Veronese, Expert, Petri, Net. Neuroscience 2019

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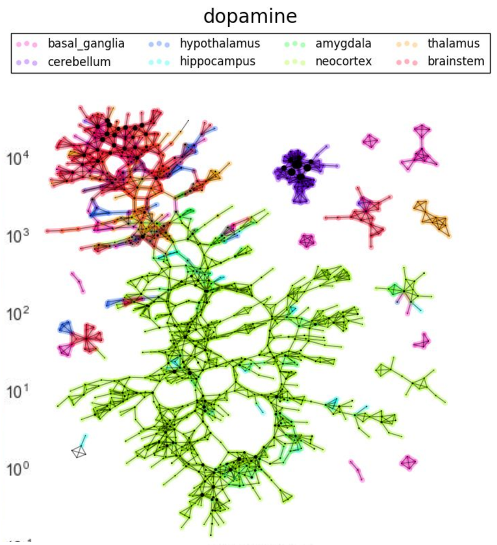
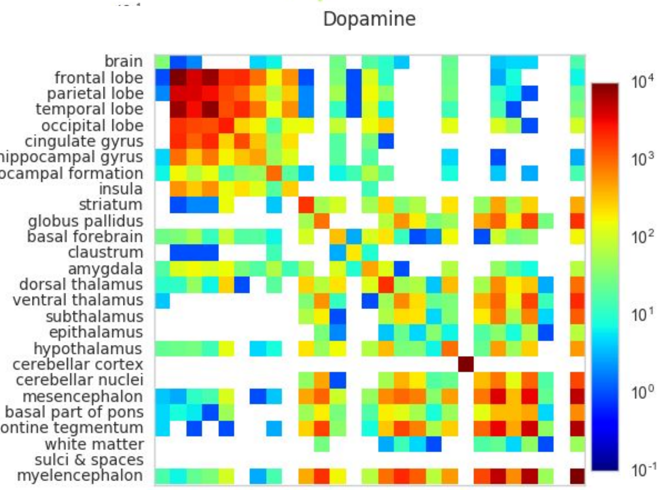
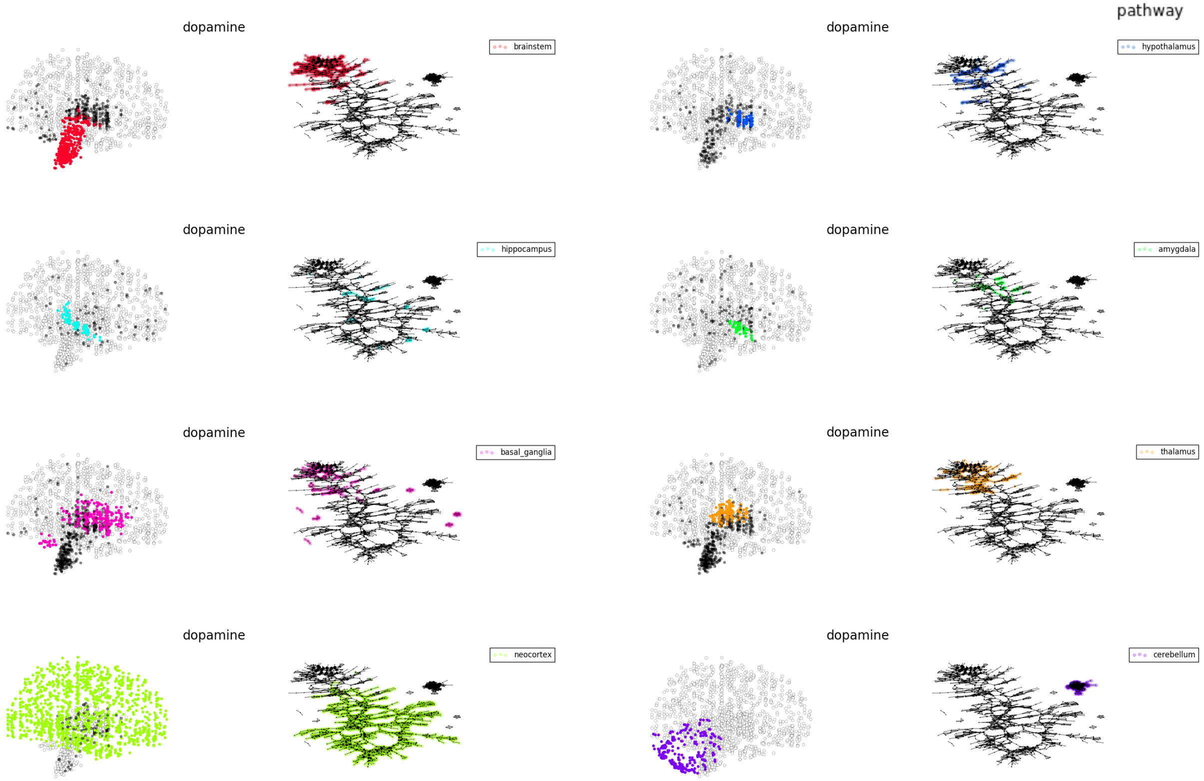
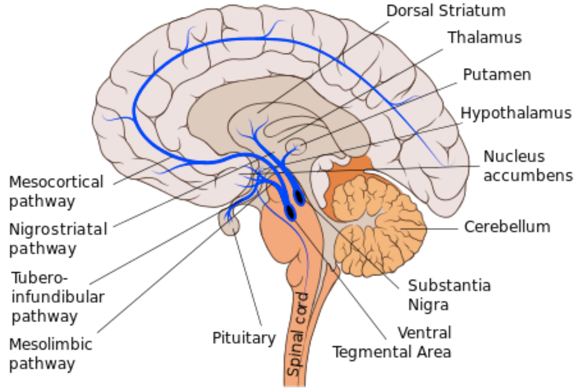
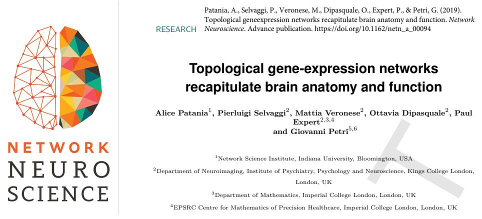










NETWORK NEUROSCIENCE

RESEARCH

Patania, A., Selvaggi, P., Veronese, M., Dipasquale, O., Expert, P., & Petri, G. (2019). Topological geneexpression networks recapitulate brain anatomy and function. *Network Neuroscience*. Advance publication. https://doi.org/10.1162/netn_a_00094

Topological gene-expression networks recapitulate brain anatomy and function

Alice Patania[1], Pierluigi Selvaggi[2], Mattia Veronese[2], Ottavia Dipasquale[2], Paul Expert[2,3,4] and Giovanni Petri[5,6]

[1]Network Science Institute, Indiana University, Bloomington, USA

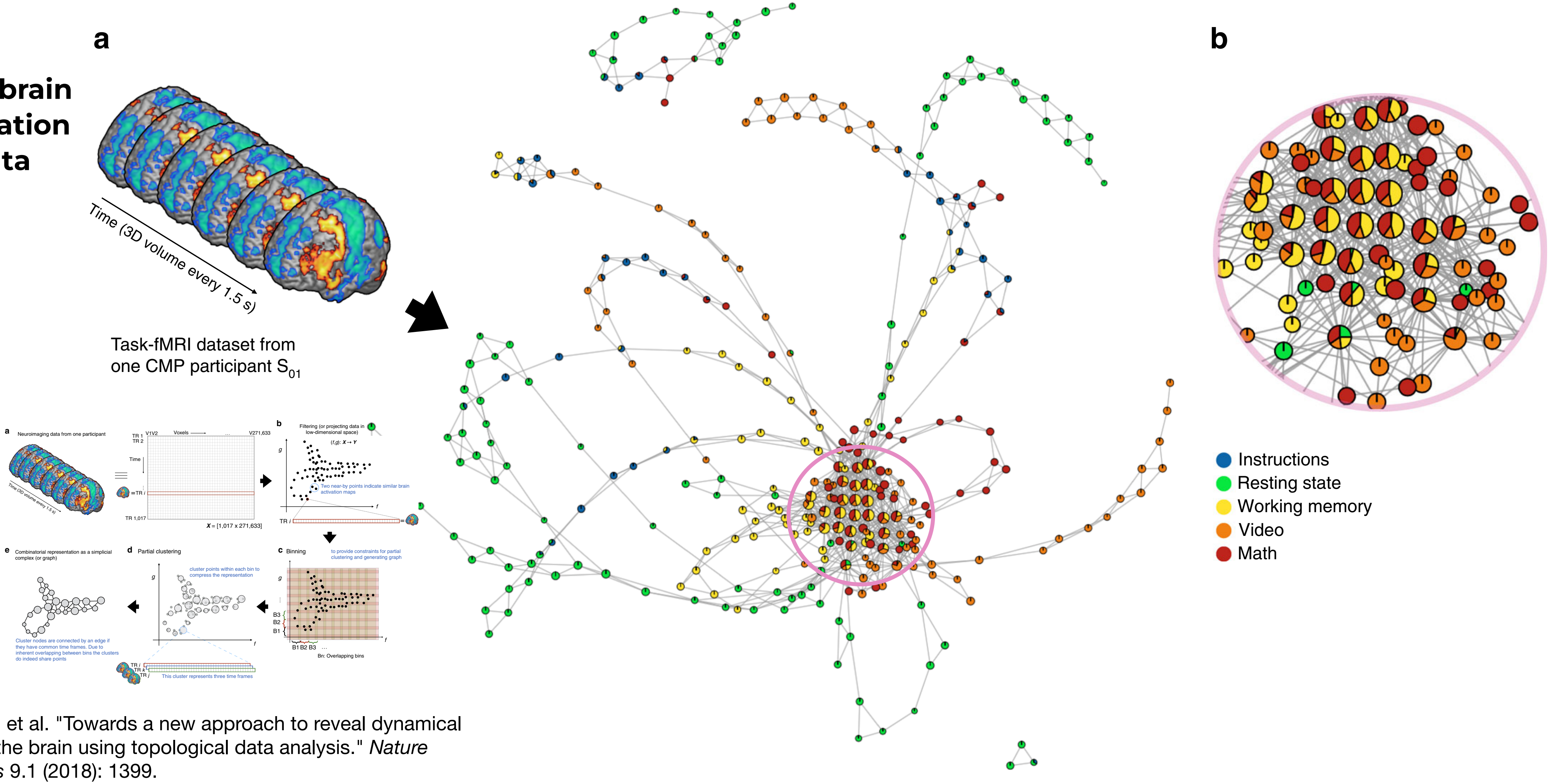
[2]Department of Neuroimaging, Institute of Psychiatry, Psychology and Neuroscience, Kings College London, London, UK

[3]Department of Mathematics, Imperial College London, London, UK

[4]EPSRC Centre for Mathematics of Precision Healthcare, Imperial College London, London, UK

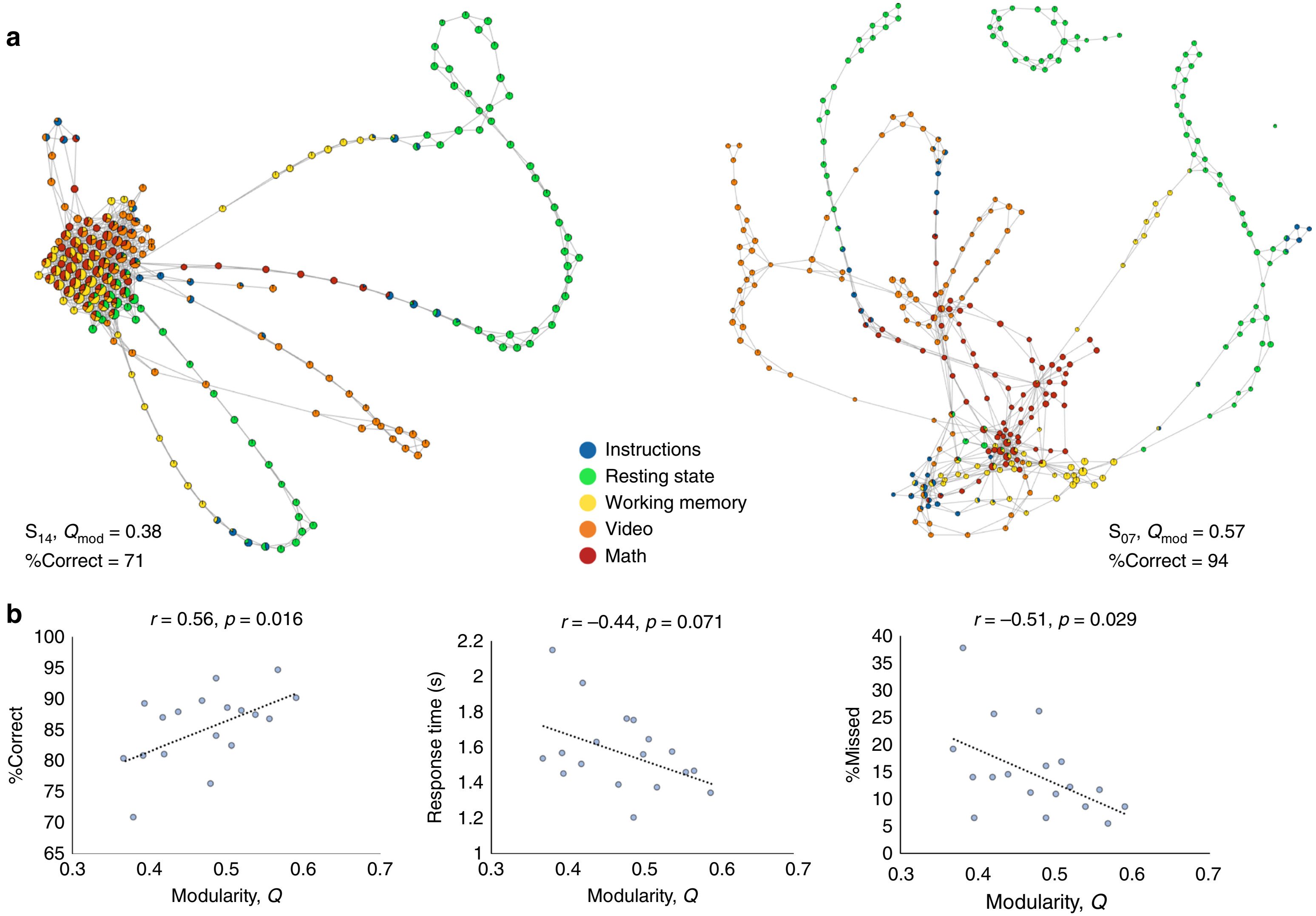
Approximate activity landscapes using topology

Task brain activation data

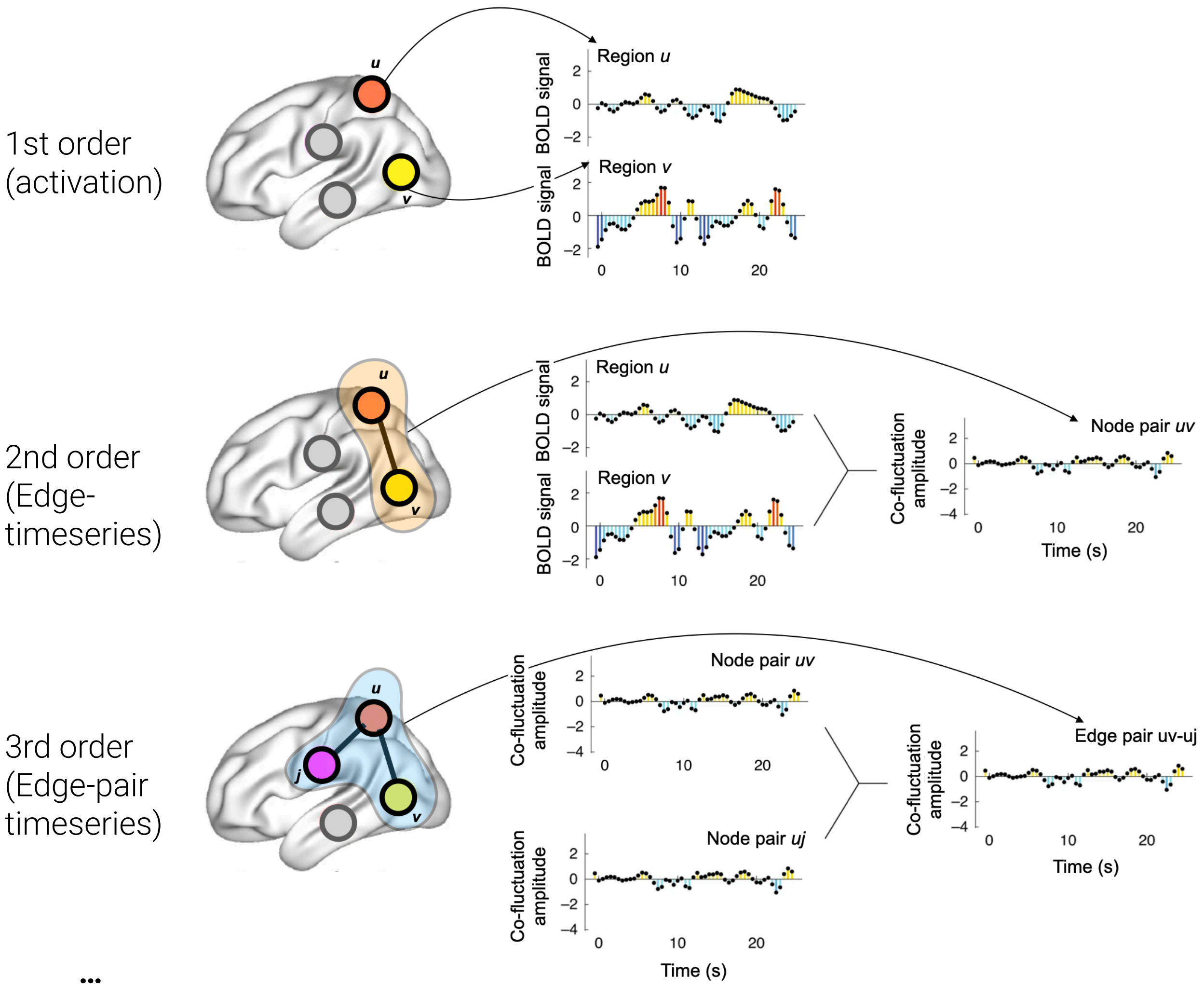


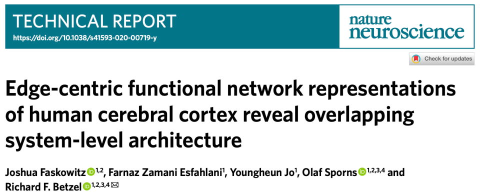
Saggar, Manish, et al. "Towards a new approach to reveal dynamical organization of the brain using topological data analysis." *Nature communications* 9.1 (2018): 1399.

Approximate activity landscapes using topology



Approximate activity landscapes using topology




TECHNICAL REPORT
https://doi.org/10.1038/s41593-020-00719-y
nature neuroscience
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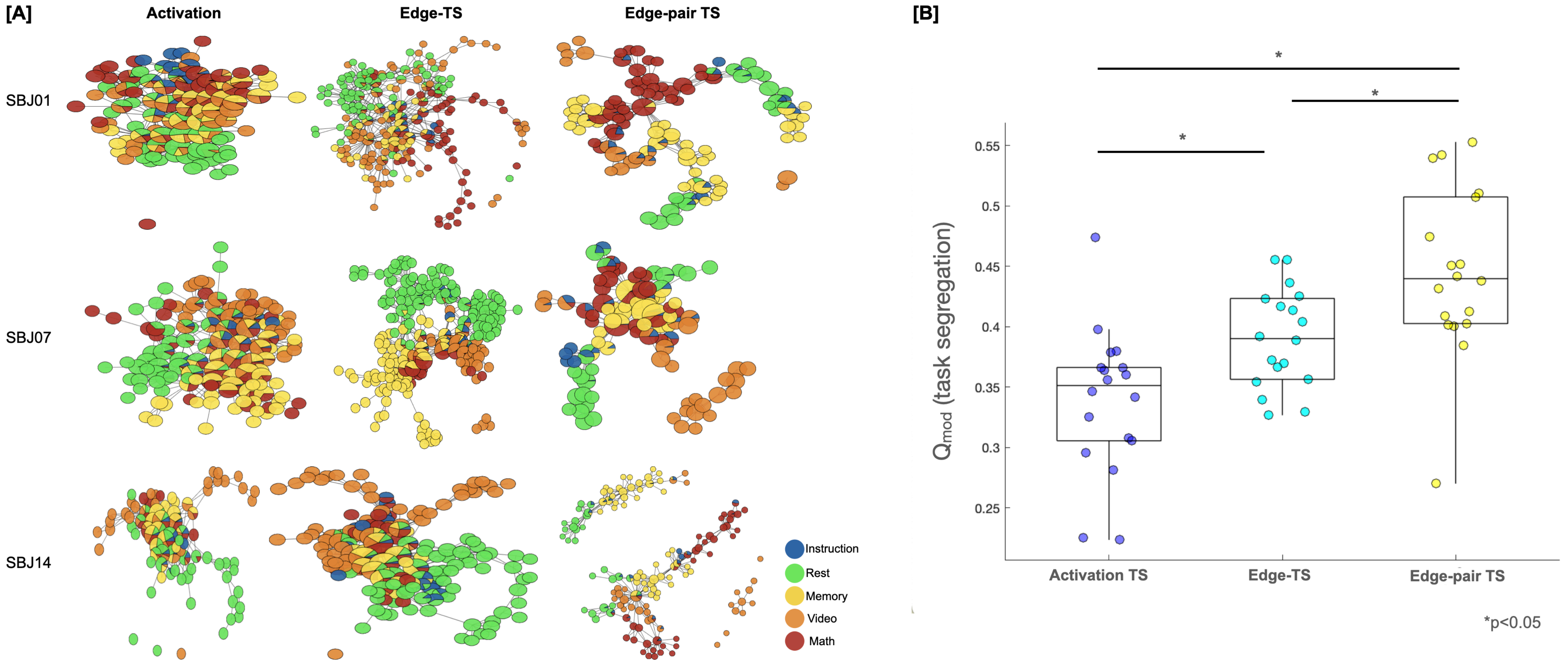
Edge-centric functional network representations of human cerebral cortex reveal overlapping system-level architecture

Joshua Faskowitz[1,2], Farnaz Zamani Esfahlani[1], Youngheun Jo[1], Olaf Sporns[1,2,3,4] and Richard F. Betzel[1,2,3,4]

...

Approximate activity landscapes using topology



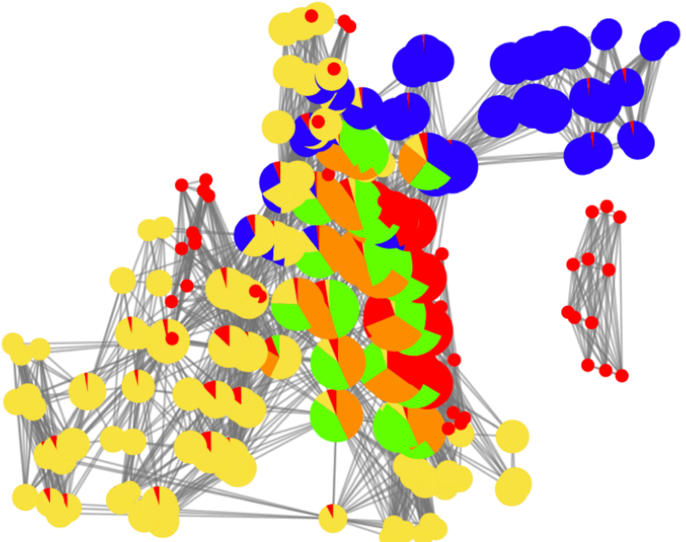
Morandini, Petri, Saggar, in prep

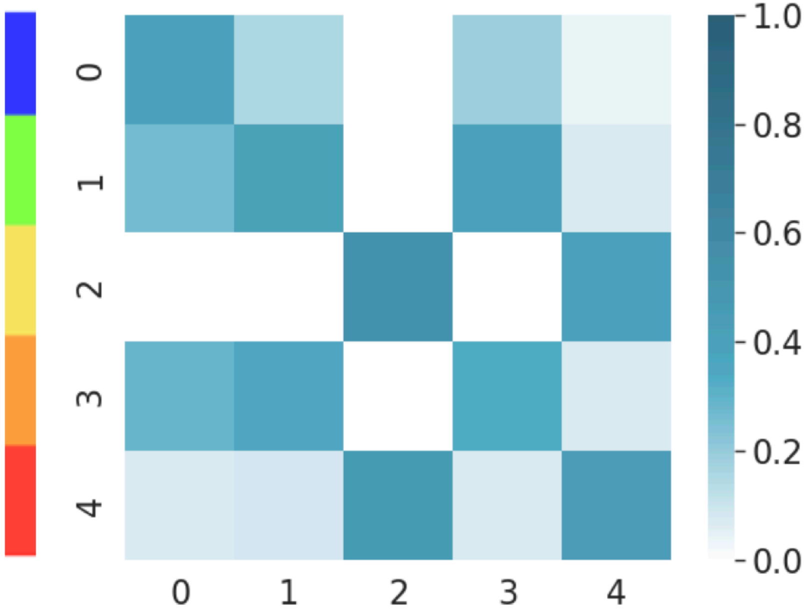
Topological fingerprinting (in general)

Def. Connectivity Mixing Matrix. Given C the number of classes:

$$\mathbf{C} = (c_{ij})_{i,j=1}^{C} \quad c_{ij} = \sum_{t_i \in i} \sum_{t_j \in j} \chi_{t_i,t_j},$$

where $\chi_{t_i,t_j} = \begin{cases} 1 & \text{if } node_{t_i} = node_{t_j} \text{ or } \exists \text{ edge}(node_{t_i}, node_{t_j}) \\ 0 & \text{otherwise} \end{cases}.$

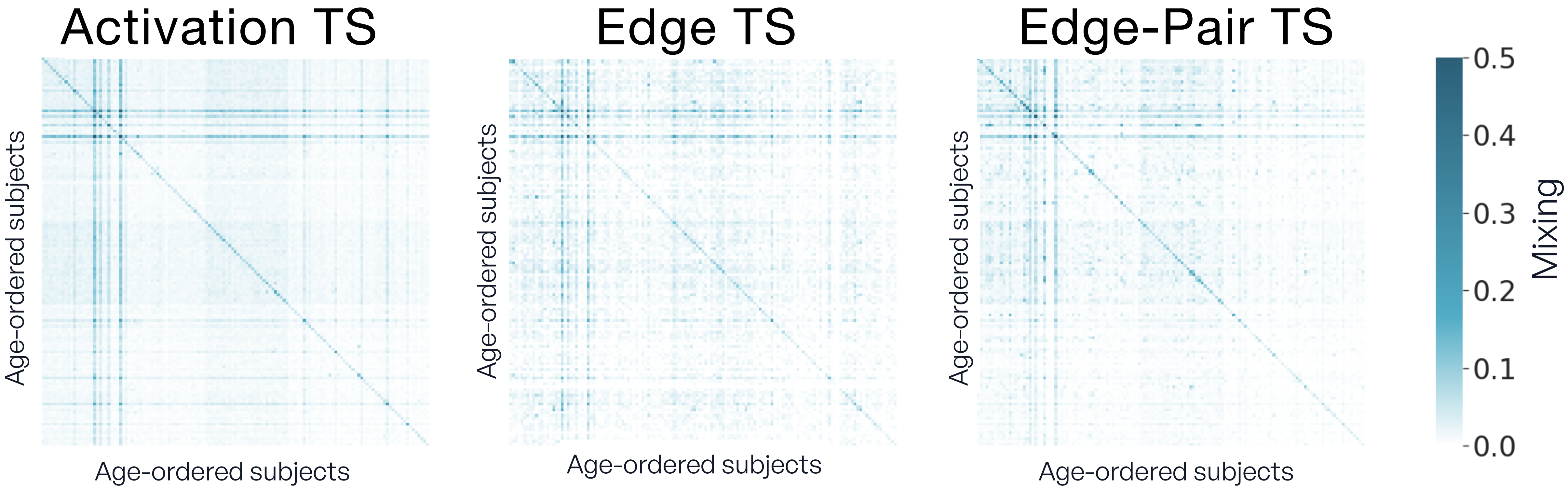




Morandini, Petri, Saggar, in prep

Topological brain fingerprinting

Is the signal strong enough across subjects?



Morandini, Petri, Saggar, in prep

Topological brain fingerprinting

Is the signal strong enough across subjects?

Intensive

Def. Mixing Matrix. Given the classes $1, \ldots, C$, assigned to each node, be M_{ij} the number of links between nodes from class i to class j. The mixing matrix of the network is

$$e = \frac{\mathbf{M}}{E},$$

where E is the total number of ordered links.

Def. Attribute assortativity coefficient. Assigned every node to a class:

$$r = \frac{Tr(e) - ||e^2||_2}{1 - ||e^2||_2}.$$

Def. Modularity. Assigned a class c_i to each node i:

$$Q = \frac{1}{2L} \sum_{i,j=1}^{N} \left(a_{ij} - \frac{k_i k_j}{2L} \right) f(c_i, c_j).$$

$$f(c_i, c_j) = D_{JS}(c_i || c_j)$$

Jensen-Shannon divergence

Discriminative

Def. Self-identifiability. Given C the number of classes and $\mathbf{C}$ the CMM:

$$I_{self}(i) = c_{ii}.$$

Def. Others-identifiability. Given C the number of classes and $\mathbf{C}$ the CMM:

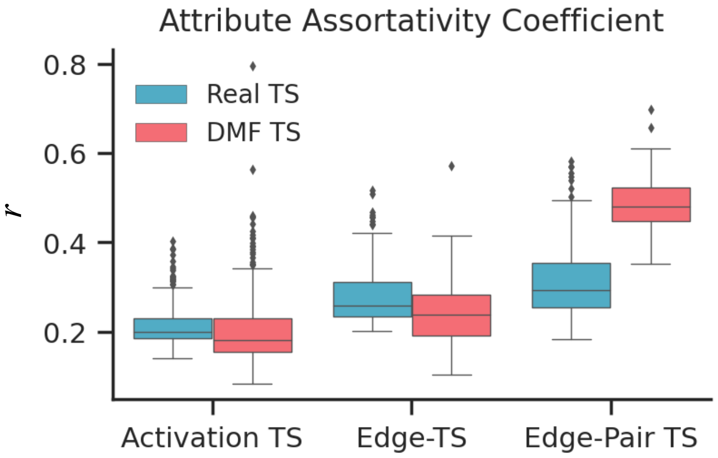
$$I_{others}(i) = \frac{1}{2} \sum_{j \neq i} (c_{ij} + c_{ji}).$$

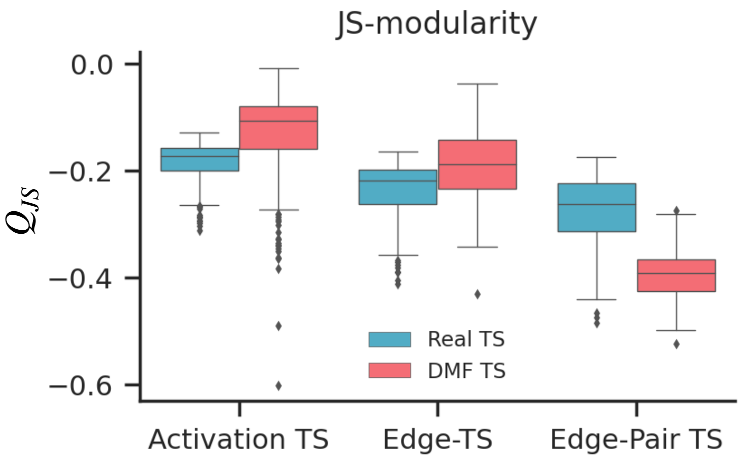
Def. Topological fingerprint. Given C the number of classes and $\mathbf{C}$ the CMM:

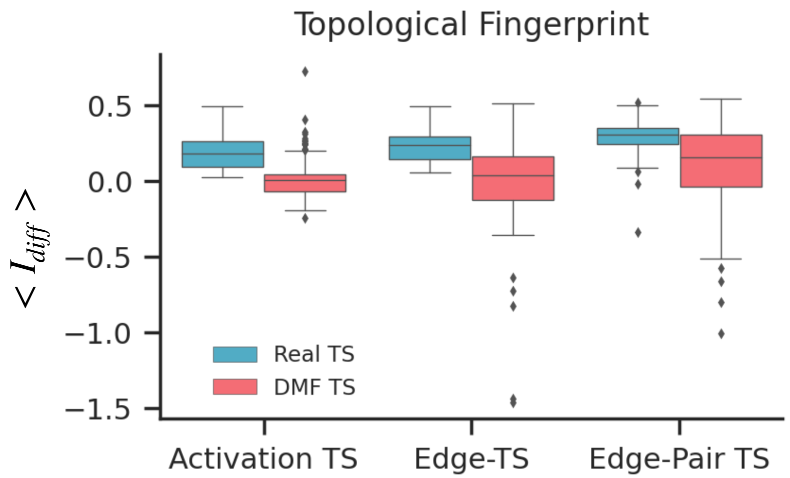
$$< I_{diff} > = \frac{< I_{self} > - < I_{others} >}{< I_{self} >},$$

where $< I_{self} >$ and $< I_{others} >$ are the average self and others identifiability.

Van De Ville, Dimitri, et al. "When makes you unique: temporality of the human brain fingerprint." *Science advances* 7.42 (2021): eabj0751.







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Topo+Info brain fingerprinting

Def. Shannon Entropy. Expected surprise of a random discrete variable X, distributed according to $p : \mathscr{X} \to [0,1]$:

$$H(X) = -\sum_{x \in \mathscr{X}} p(x)\, log(p(x)).$$

Def. Joint Entropy. Expected surprise of a set of random discrete variables $\mathbf{X} = X_1, X_2, \ldots, X_n$, distributed according to $p_i : \mathscr{X}_i \to [0,1], i = 1, \ldots, n$:

$$H(\mathbf{X}) = H(X_1, \ldots, X_n) = -\sum_{x_i \in \mathscr{X}_i,\, i=1,\ldots,n} p(x_1, \ldots x_n)\, log(p(x_1, \ldots, x_n)).$$

Def. Ω-information.

$$\Omega(\mathbf{X}) = \Omega(X_1, \ldots, X_n) = (n-2)H(\mathbf{X}) - \sum_{i=1}^{n} (H(X_i) - H(\mathbf{X}_{-i})).$$

$\Omega(\mathbf{X}) > 0$

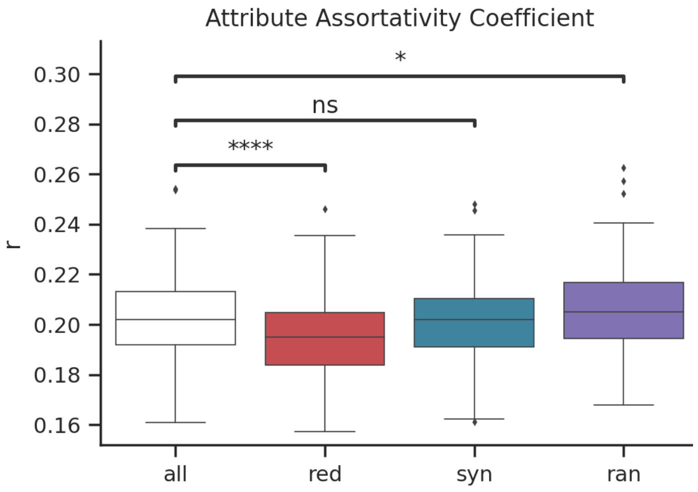
REDUNDANCY

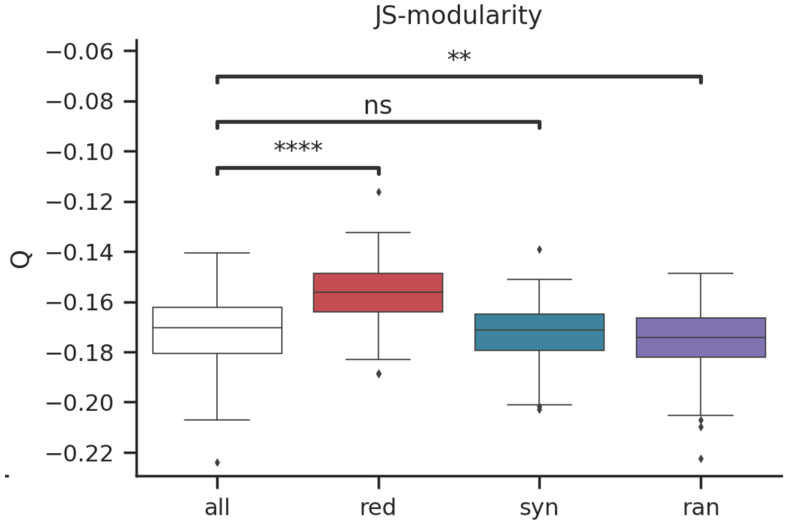
$\Omega(\mathbf{X}) < 0$

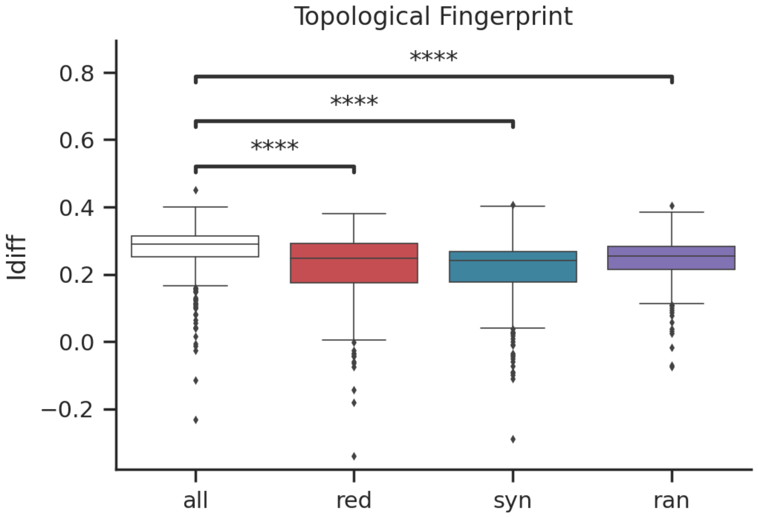
SYNERGY

Intensive

Discriminative







Morandini, Petri, Saggar, in prep

Topo+Info brain fingerprinting

Summing up

- Topological information (simplification) discriminates well across individuals
- Stronger effect for higher-order timeseries
- Global markers, but no relation to the actual synergy/redundancy patterns

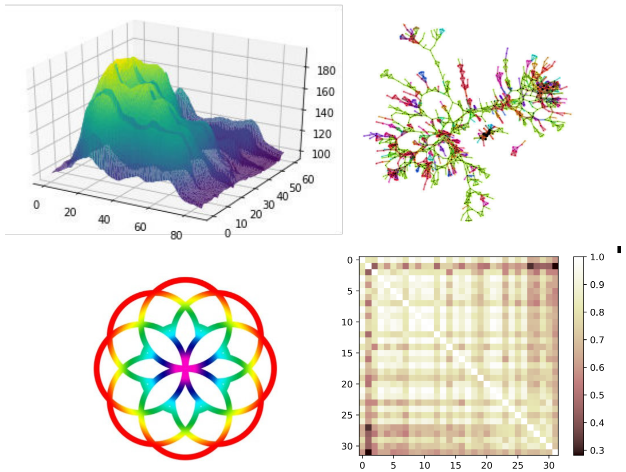
Can topology quantify local shapes?

Functional, structural, you name it…

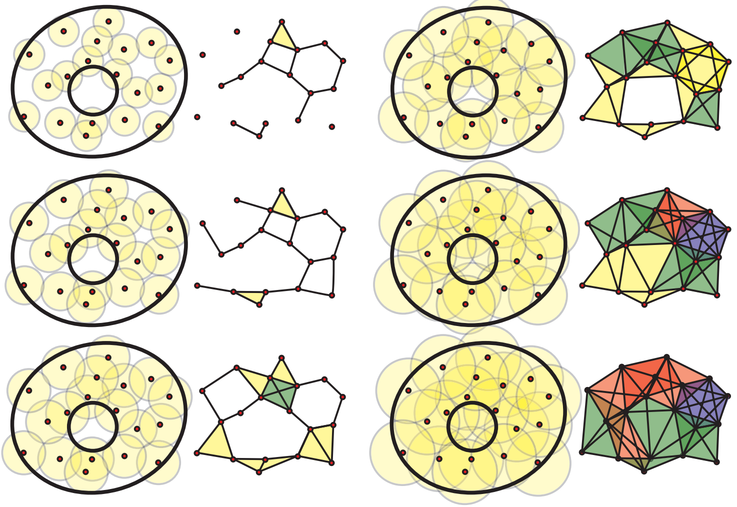
What does it mean in practice?

Persistent homology pipeline (Ghrist 2008)

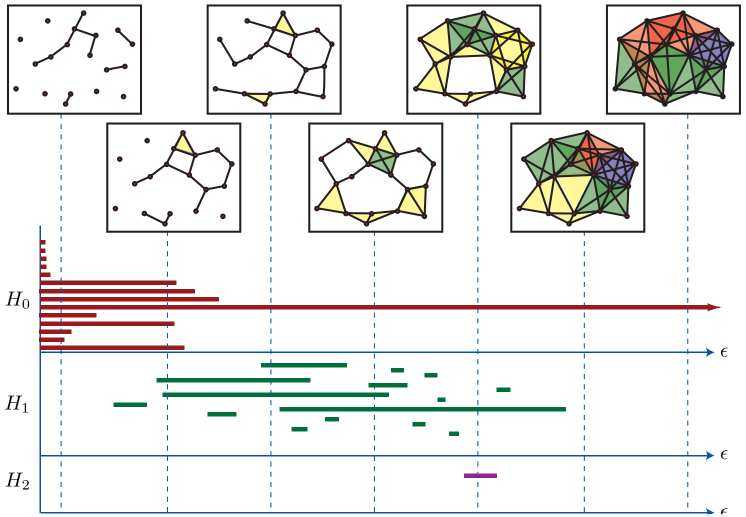
Data of sorts



Filtration over distance/density/weights



Homological properties



Mapper Pipeline (Singh et al 2007)

Point cloud

colouring (projecting)
using geometric filters

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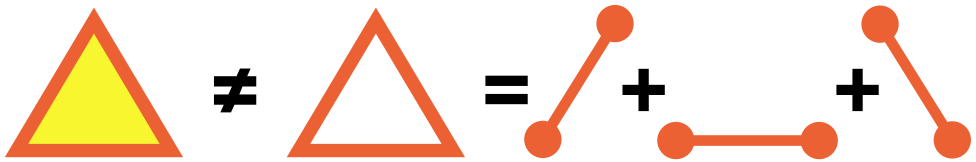
Clustering and
network construction binning

From data to simplices

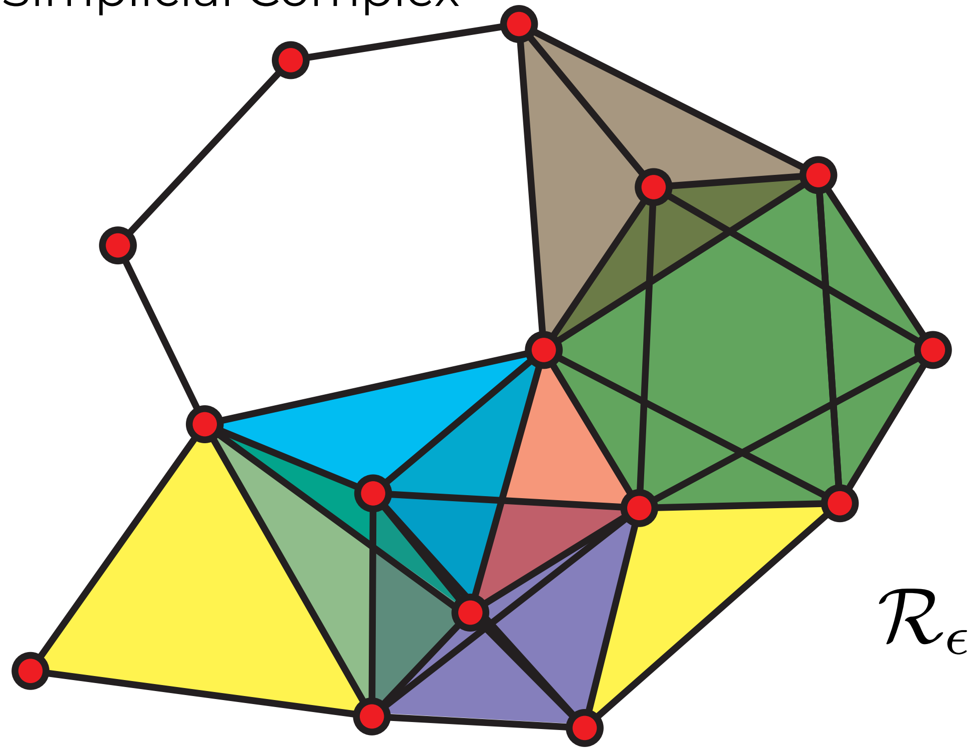
DOT
= 0-simplex

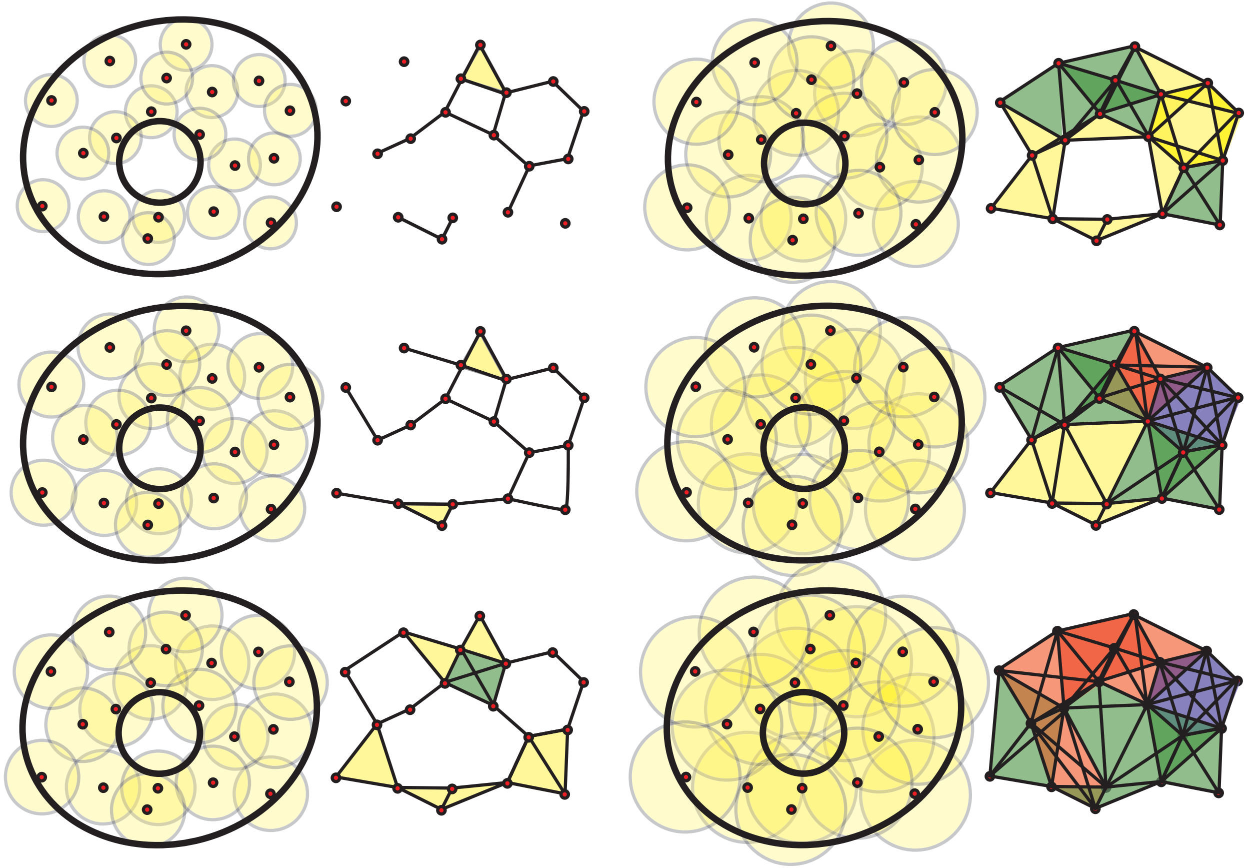
EDGE =
1-simplex

TRIANGLE
= 2-simplex



Simplicial Complex





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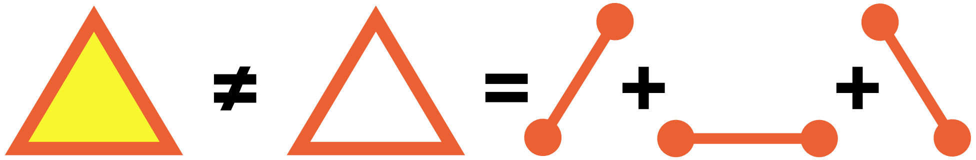
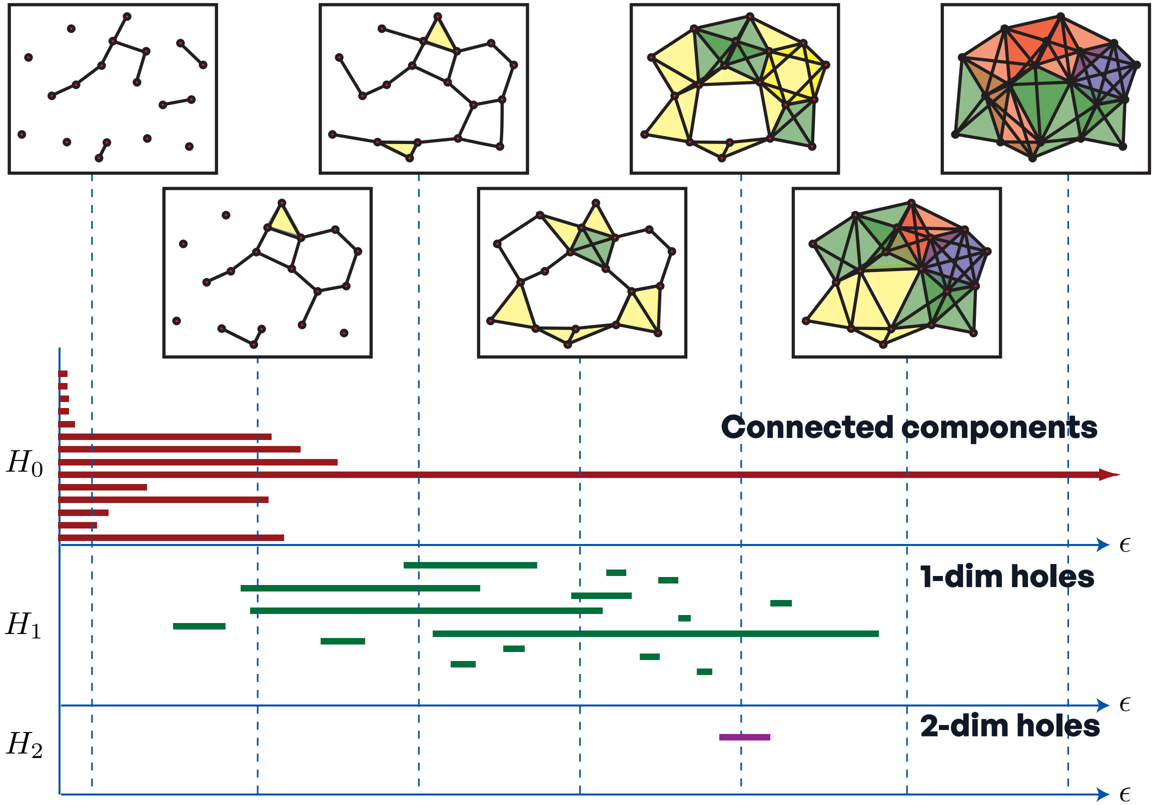
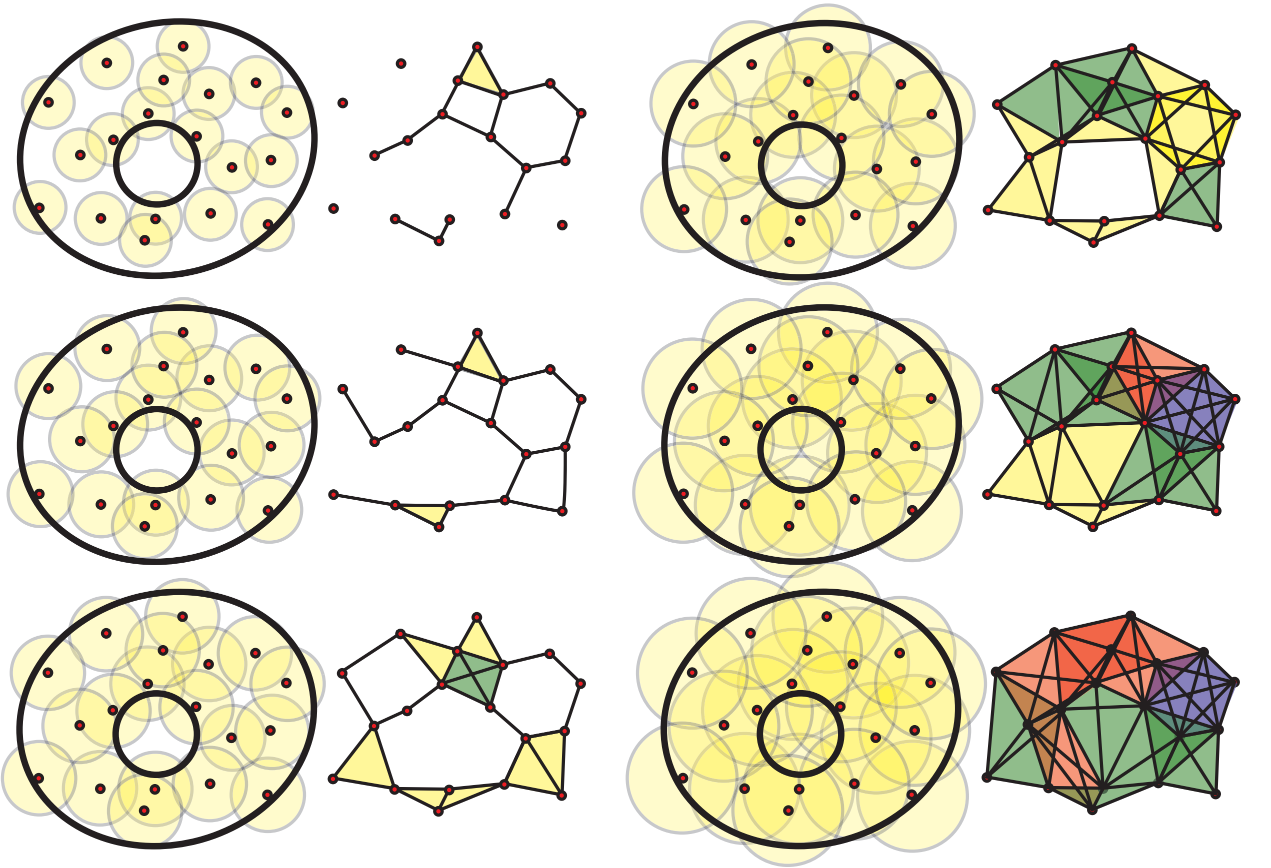
= 2-simplex 





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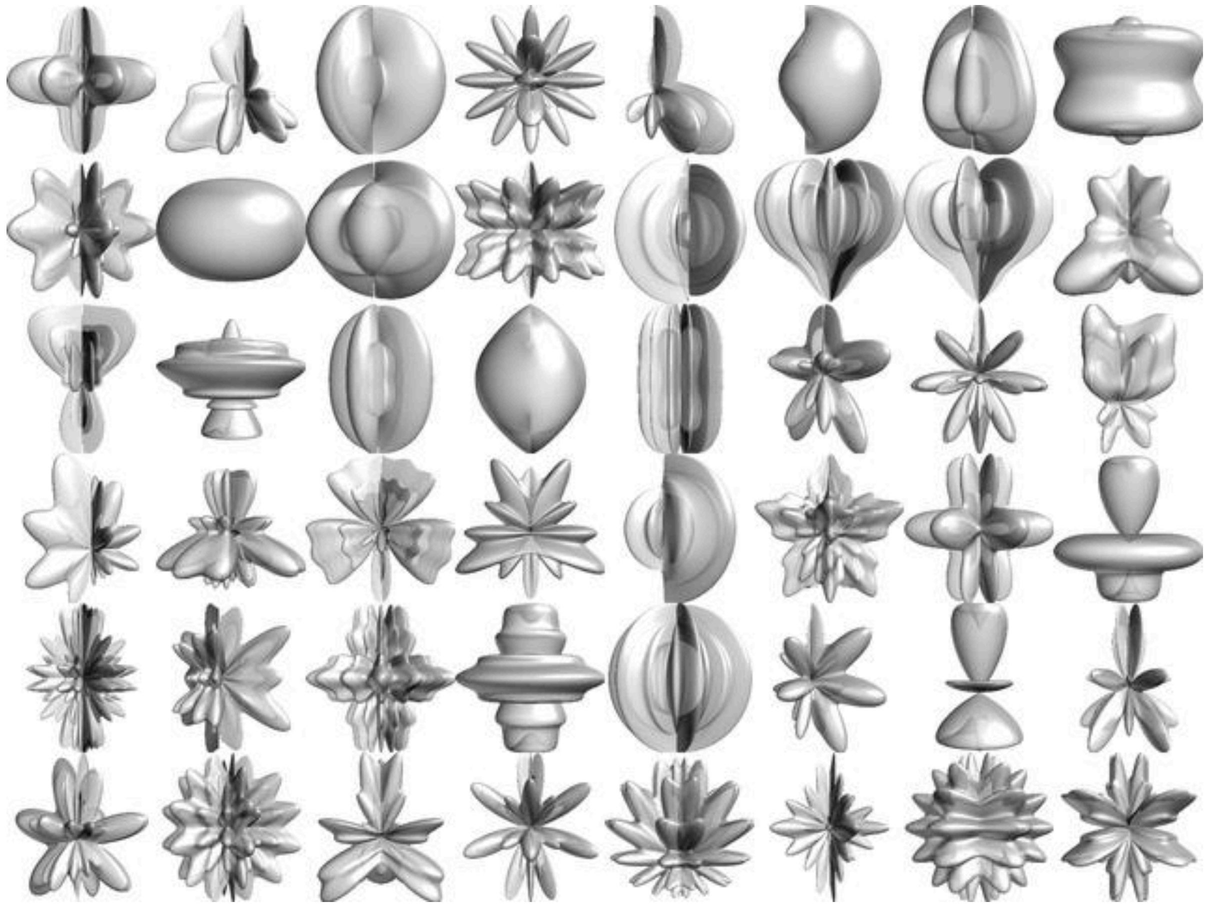
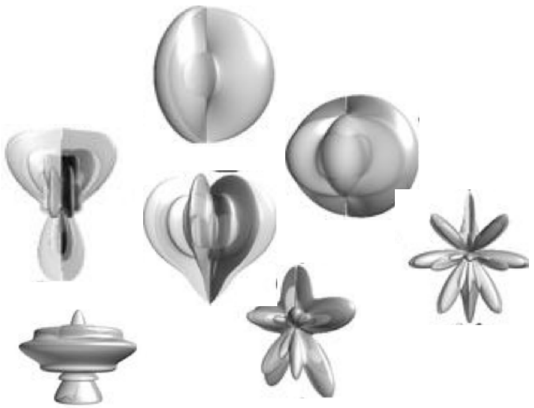
Quantitative topological comparison



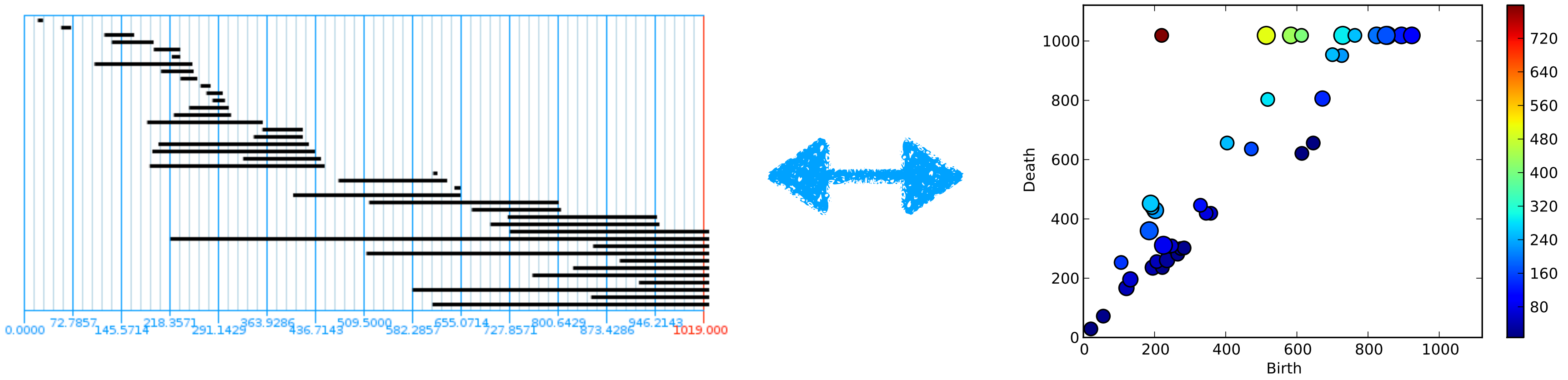
Quantitative topological comparison

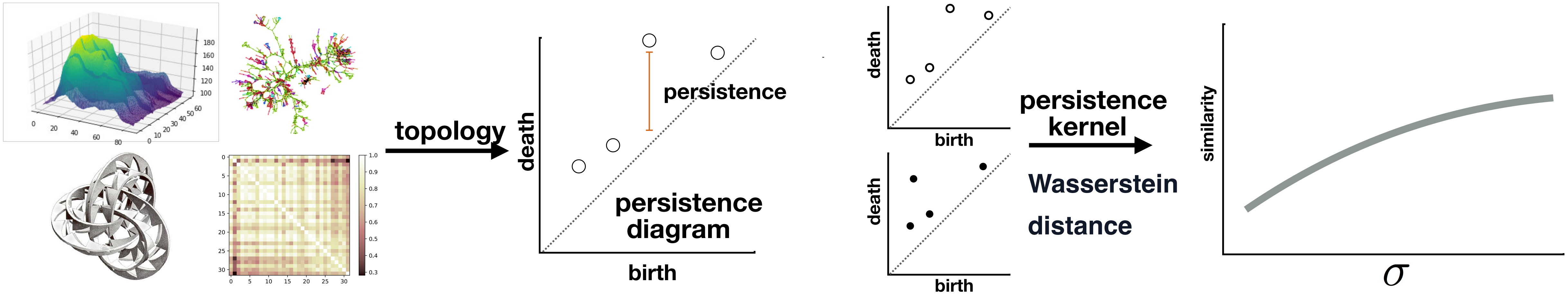


Quantitative topological comparison

Aktas, Mehmet E., Esra Akbas, and Ahmed El Fatmaoui. "Persistence homology of networks: methods and applications." *Applied Network Science* 4.1 (2019): 1-28.
Fasy, Brittany, et al. "Comparing distance metrics on vectorized persistence summaries." *TDA {\&} Beyond*. 2020.
Chung, Moo K., et al. "Topological distances between brain networks." *Connectomics in NeuroImaging: First International Workshop, CNI 2017, Held in Conjunction with MICCAI 2017, Quebec City, QC, Canada, September 14, 2017, Proceedings 1*. Springer International Publishing, 2017.

Quantitative topological comparison

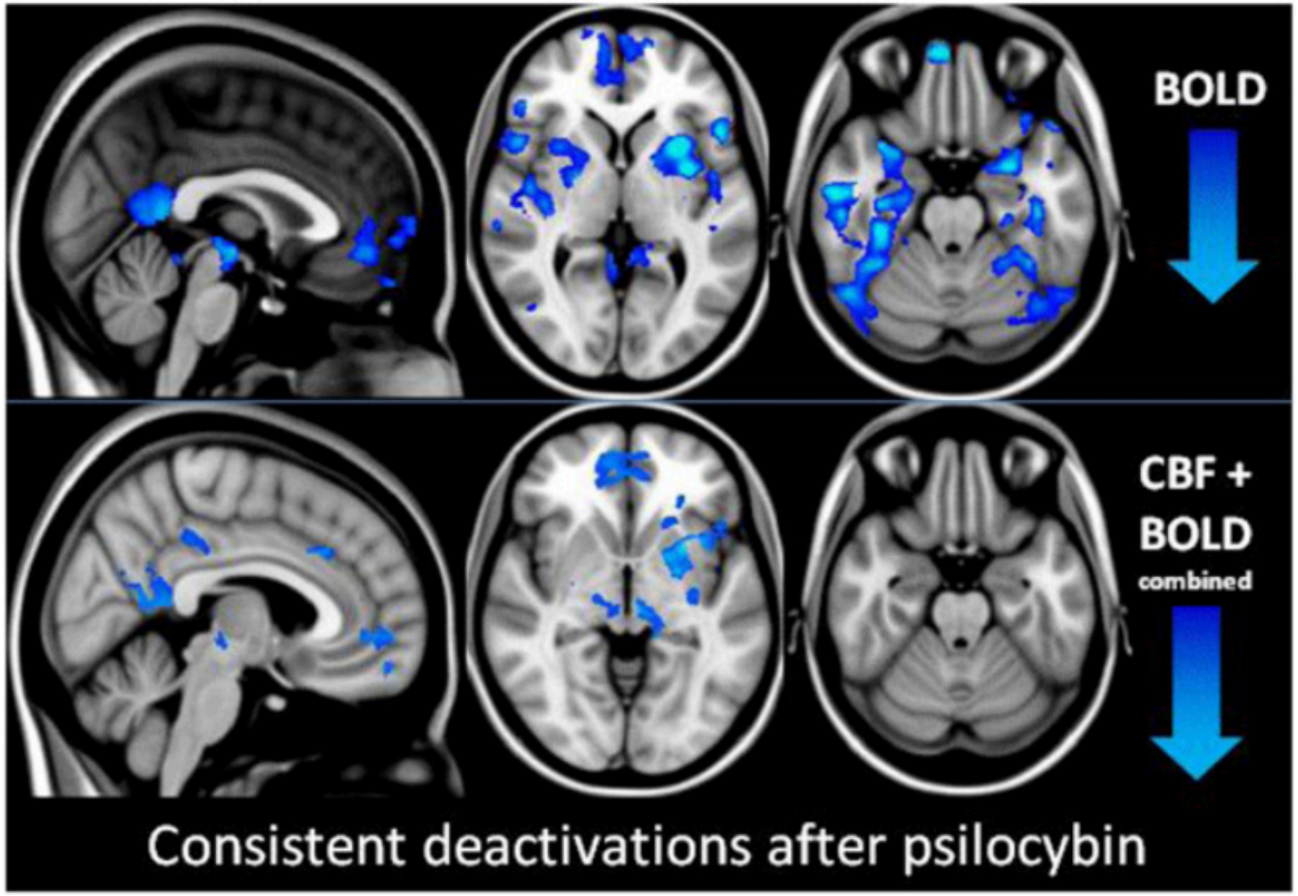




BrainZ

Altered functional topology

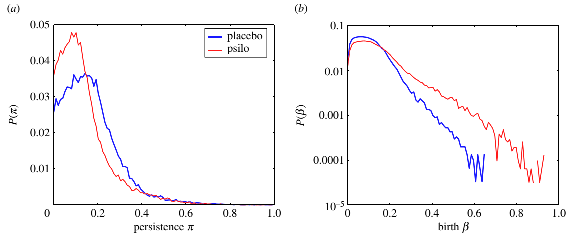
rs-fMRI
15 subjects,2 sessions
1 recording condition

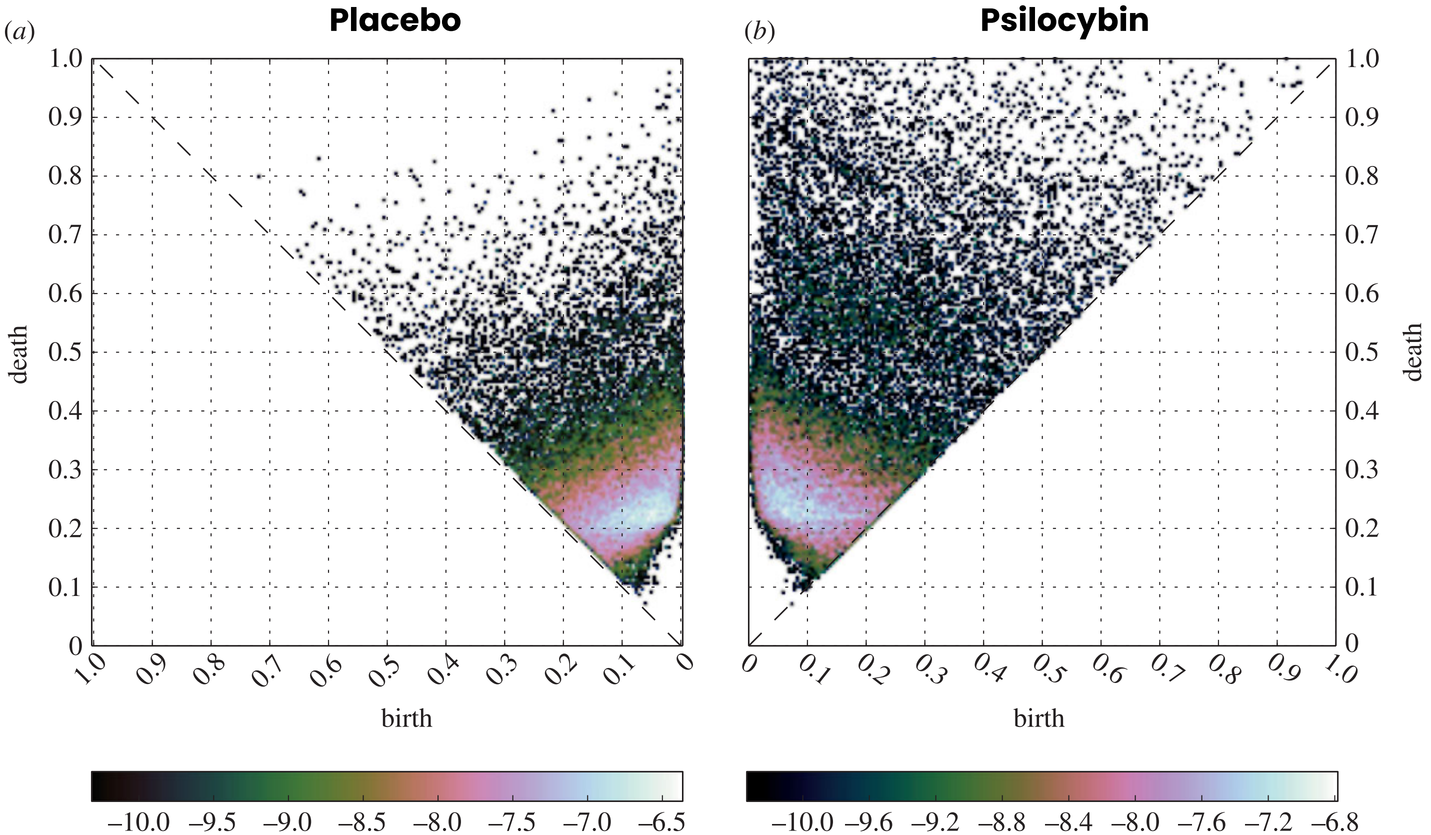


Carhart-Harris, Robin L., et al. "Neural correlates of the psychedelic state as determined by fMRI studies with psilocybin." *Proceedings of the National Academy of Sciences* 109.6 (2012): 2138-2143.

Petri, Giovanni, et al. "Homological scaffolds of brain functional networks." *Journal of The Royal Society Interface* 11.101 (2014): 20140873.

Altered functional topology





rs-fMRI
15 subjects,2 sessions
1 recording condition

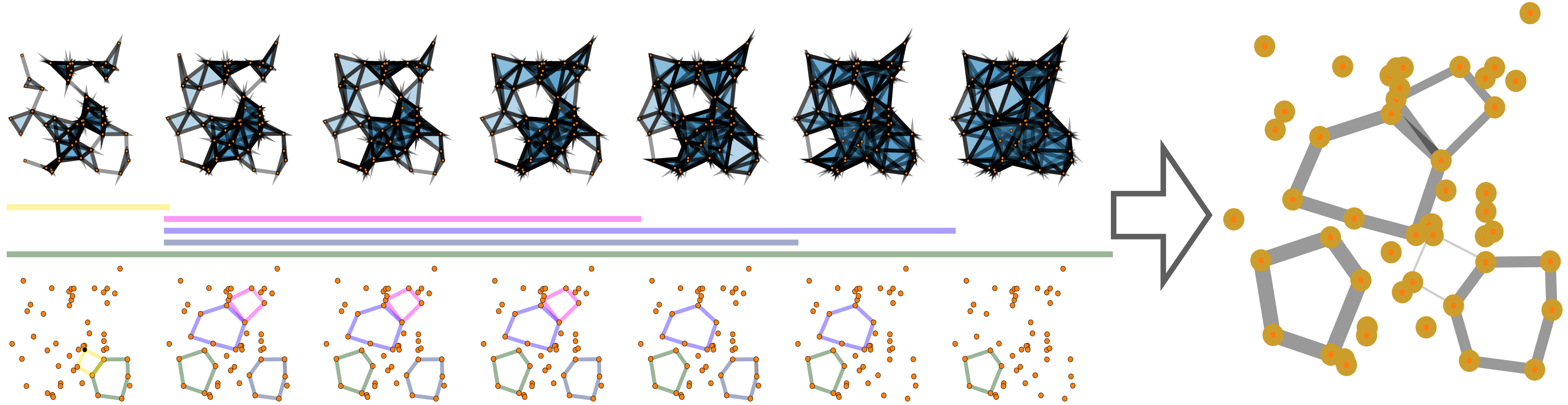
Carhart-Harris, Robin L., et al. "Neural correlates of the psychedelic state as determined by fMRI studies with psilocybin." *Proceedings of the National Academy of Sciences* 109.6 (2012): 2138-2143.

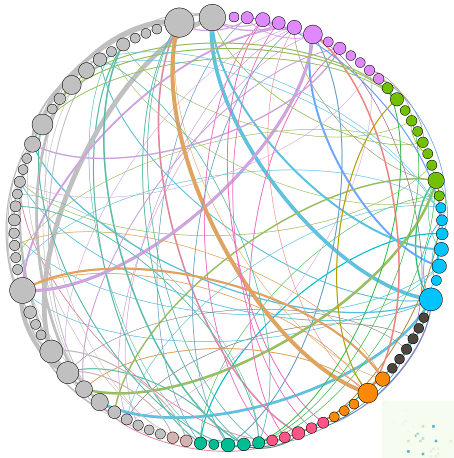
Group level persistence diagrams

Localisation of information?

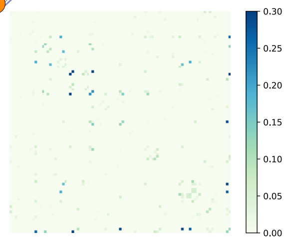
Petri, Giovanni, et al. "Homological scaffolds of brain functional networks." *Journal of The Royal Society Interface* 11.101 (2014): 20140873.

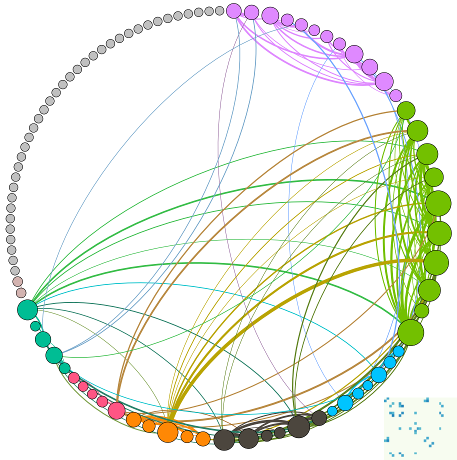
Scaffolds in one slide



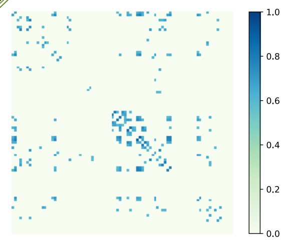


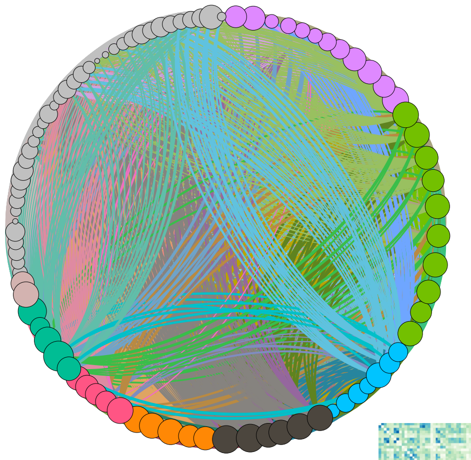
Scaffold



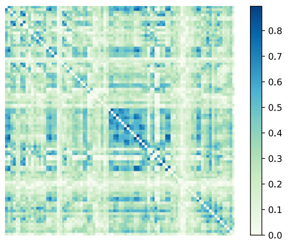


Diluted FC



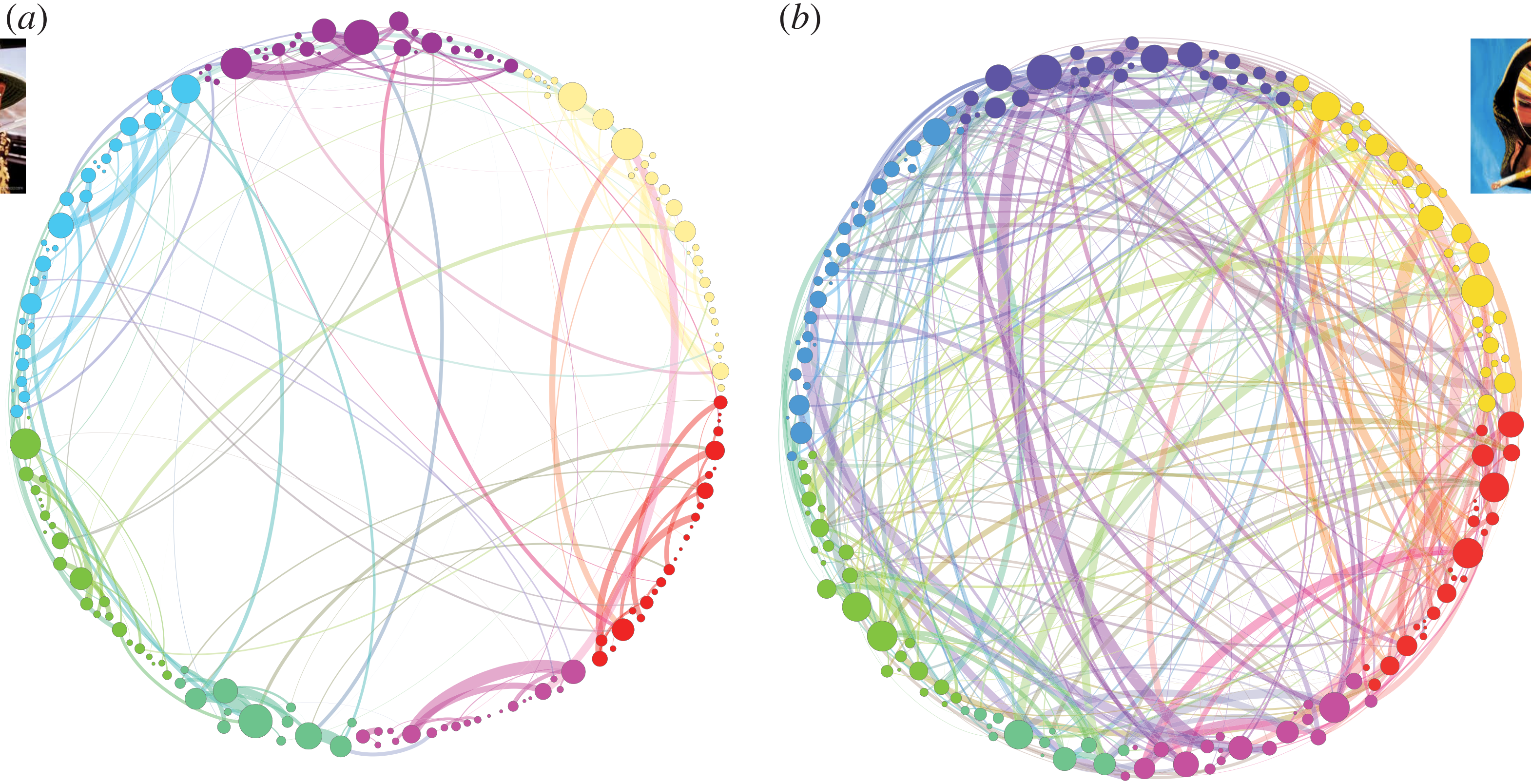


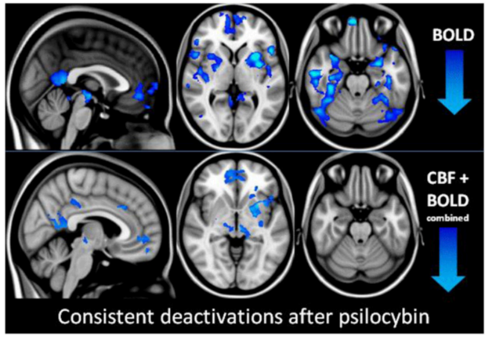
Min-weight FC



Brain scaffolds: local alterations

distributed

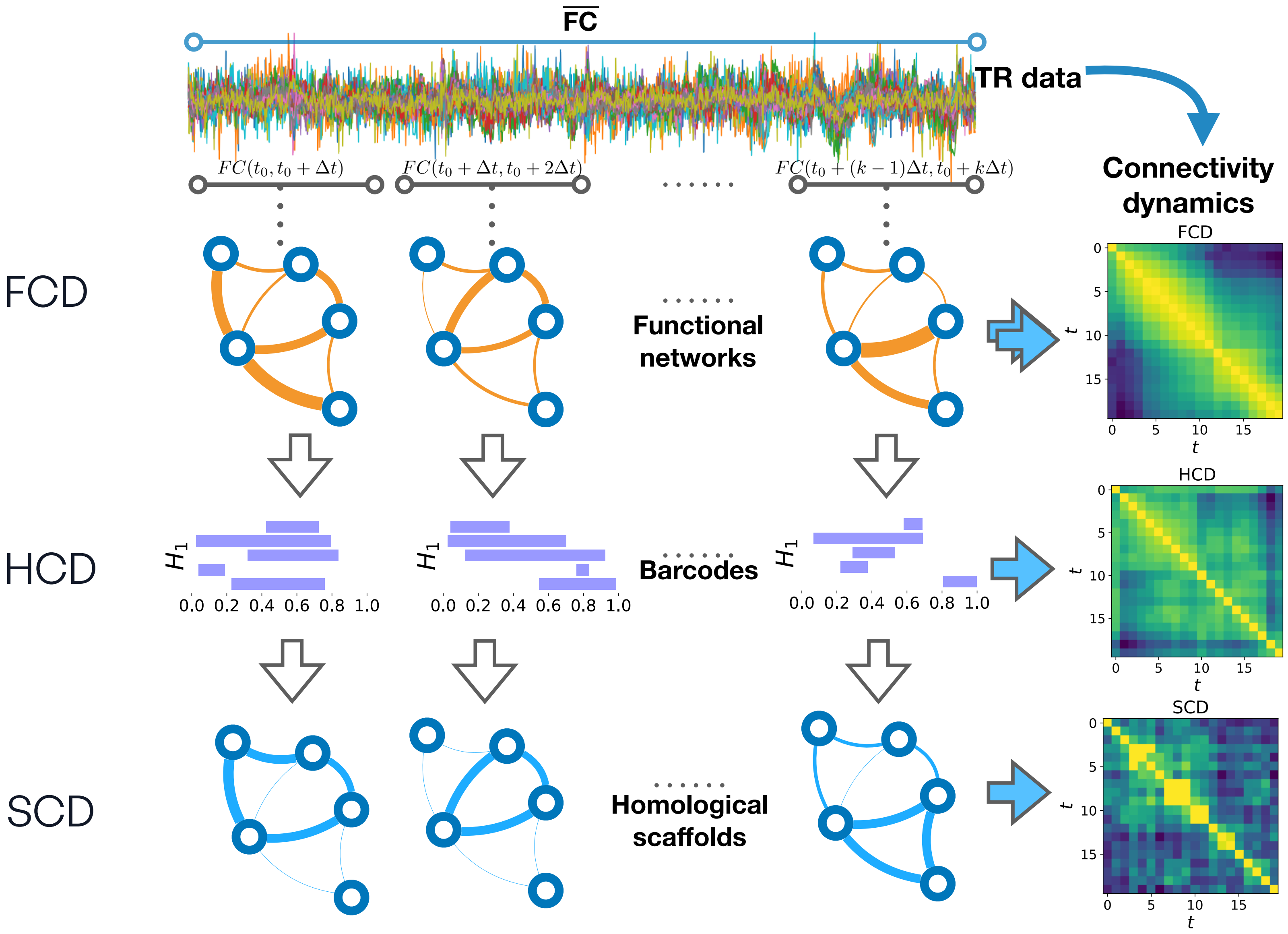




Distributed reorganisation of the hierarchy of functional circuits

Petri, Giovanni, et al. "Homological scaffolds of brain functional networks." *Journal of The Royal Society Interface* 11.101 (2014): 20140873.

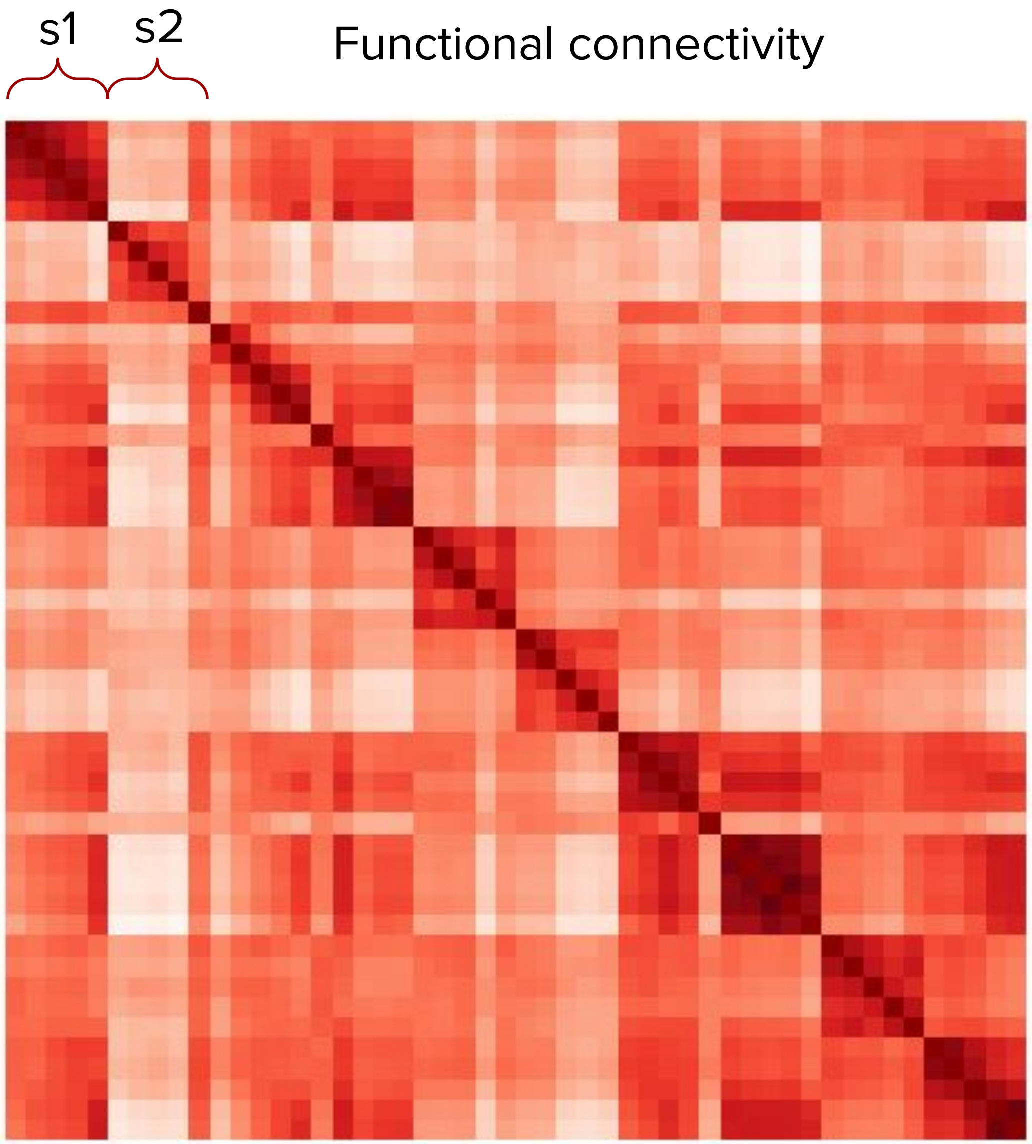
Scaffold fingerprinting

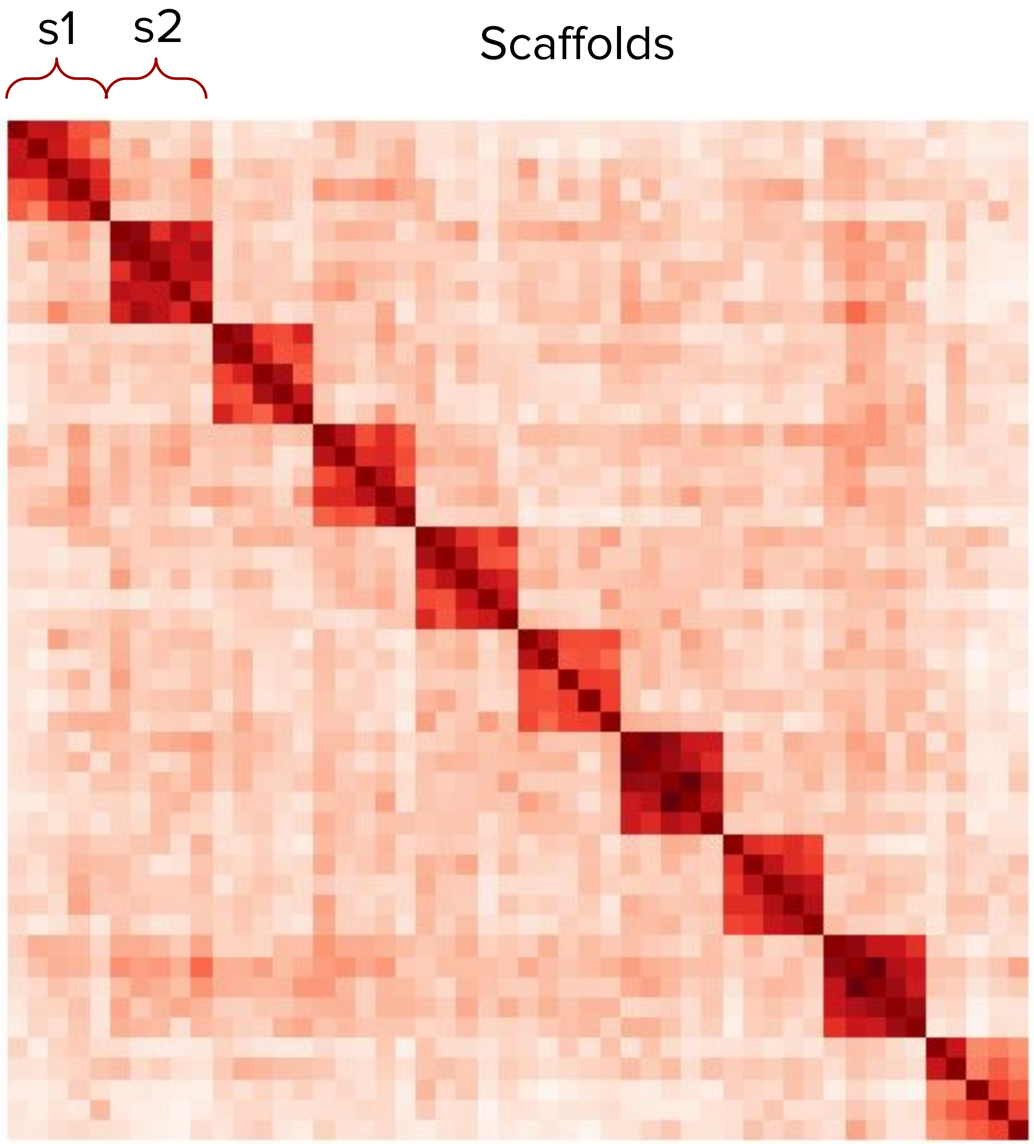


Poetto, Saggar, Battaglia, Vaccarino, Rabuffo, Petri in prep

Scaffold fingerprinting

100 subjects (HCP), rs-fMRI, test+retest



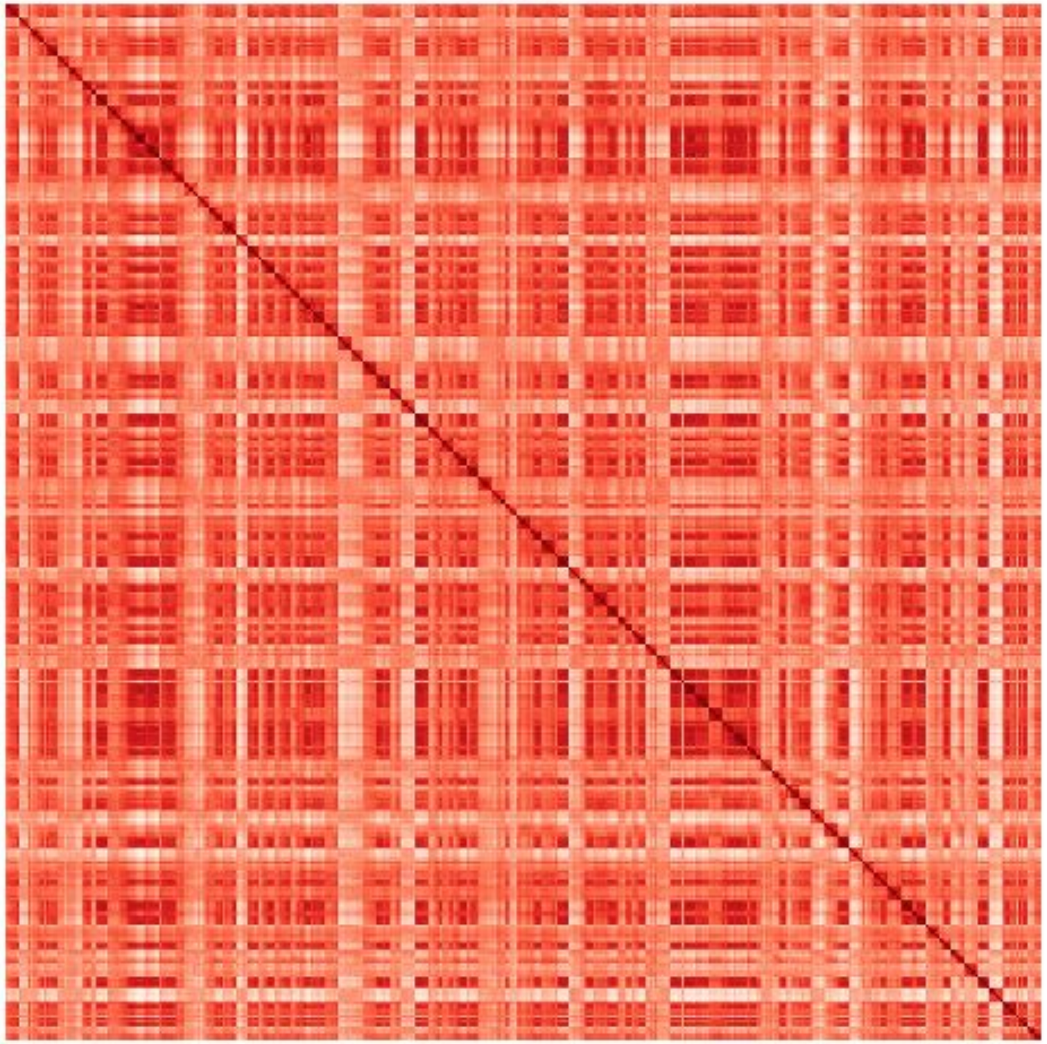


Poetto, Saggar, Battaglia, Vaccarino, Rabuffo, Petri in prep

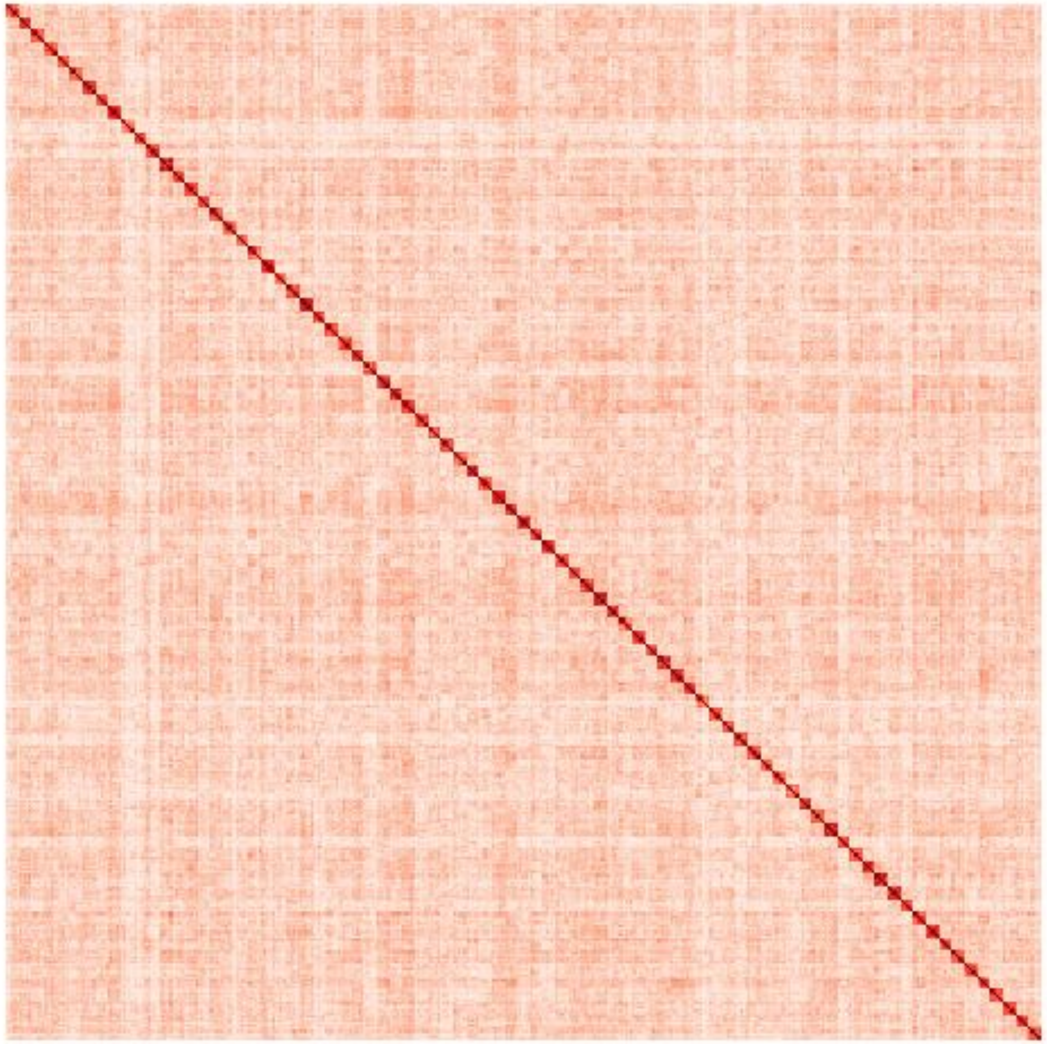
Scaffold fingerprinting

100 subjects (HCP), rs-fMRI, test+retest

Functional connectivity



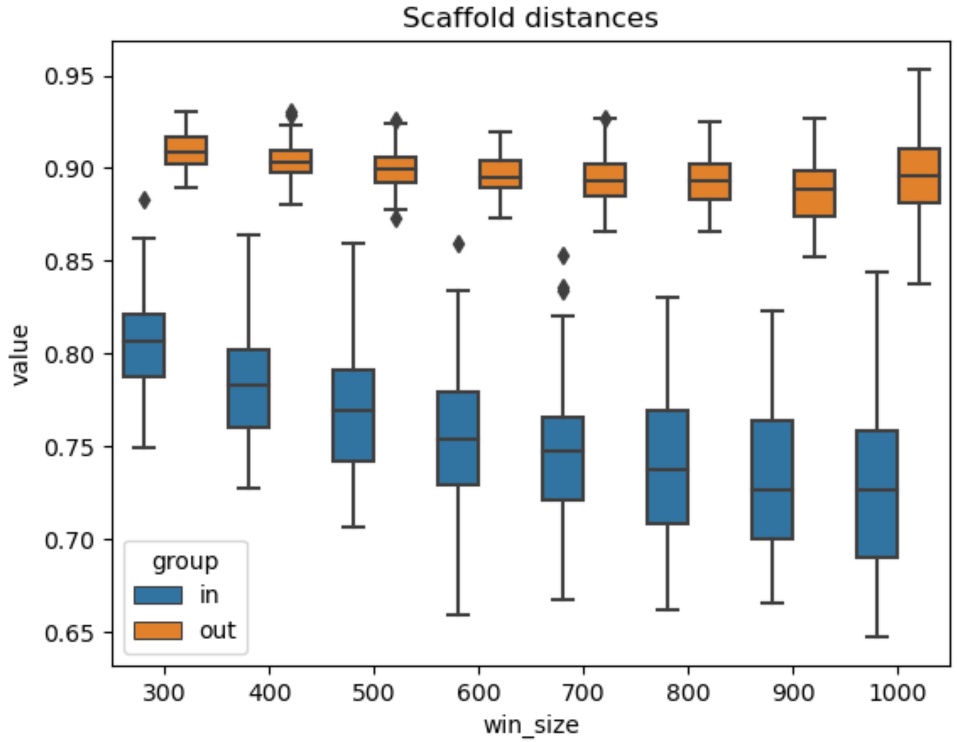
Scaffolds

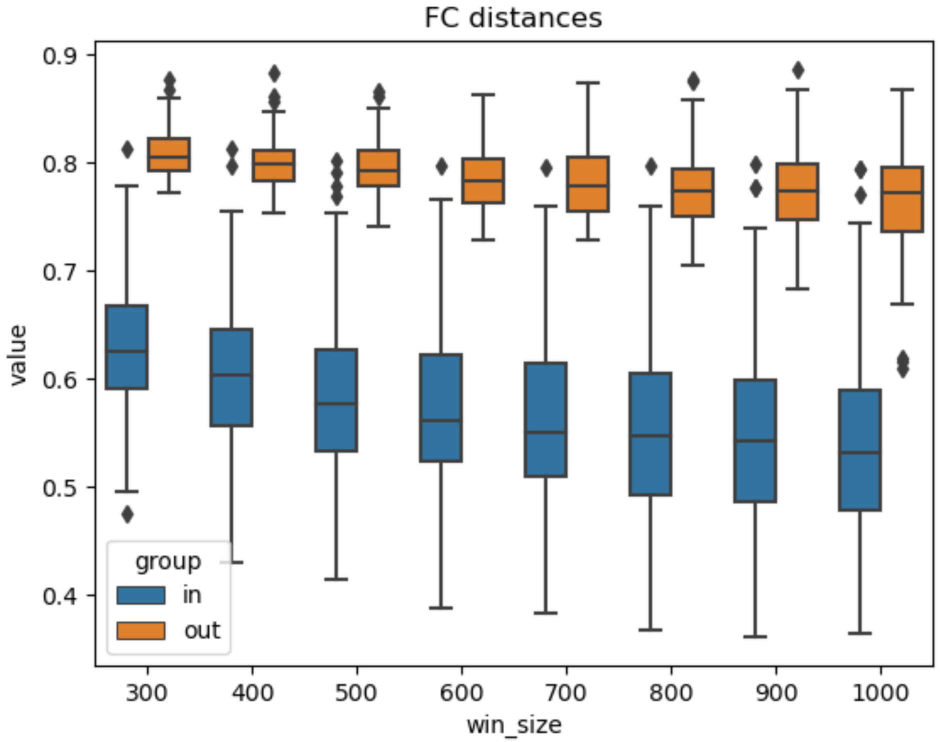


Poetto, Saggar, Battaglia, Vaccarino, Rabuffo, Petri in prep

Scaffold fingerprinting

100 subjects (HCP), rs-fMRI, test+retest





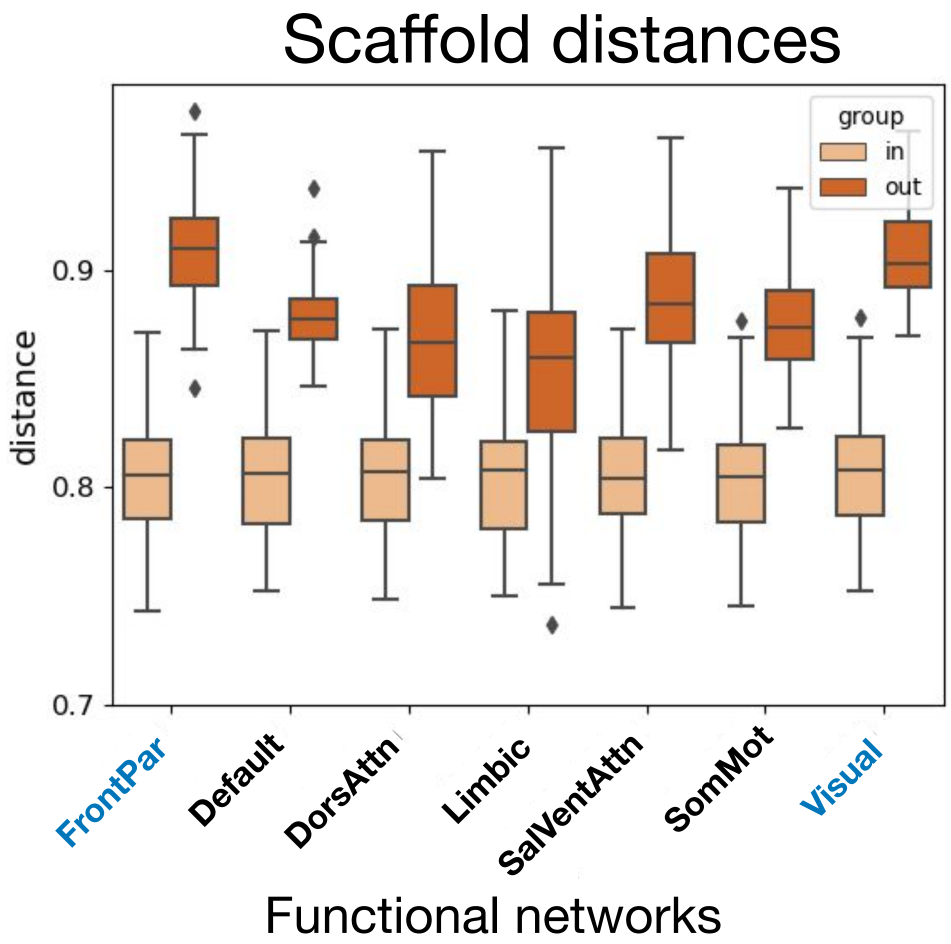
Incredibile fingerprinting capacity!

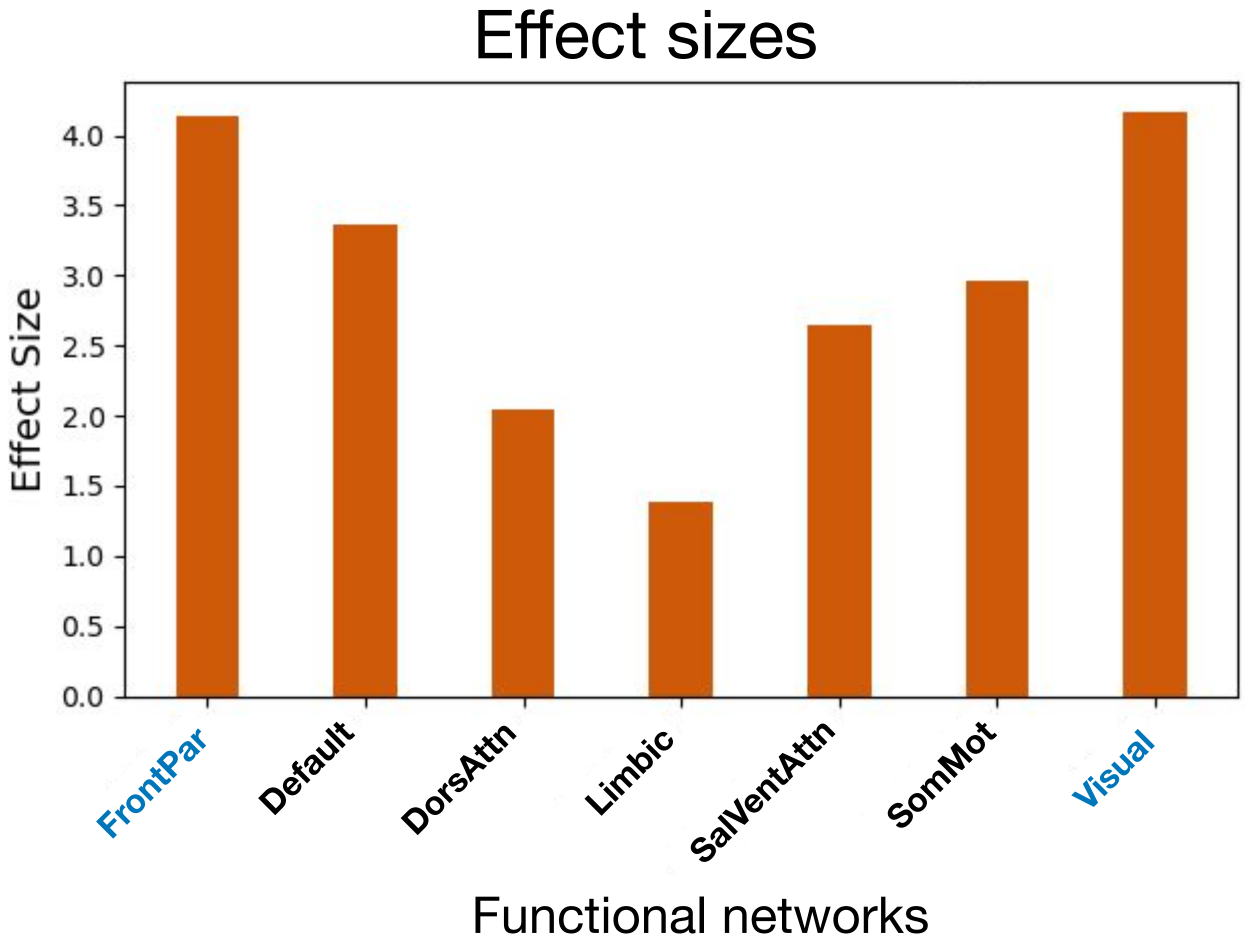
QUASI idea on the origin!

Poetto, Saggar, Battaglia, Vaccarino, Rabuffo, Petri in prep

Scaffold fingerprinting

100 subjects (HCP), rs-fMRI, test+retest





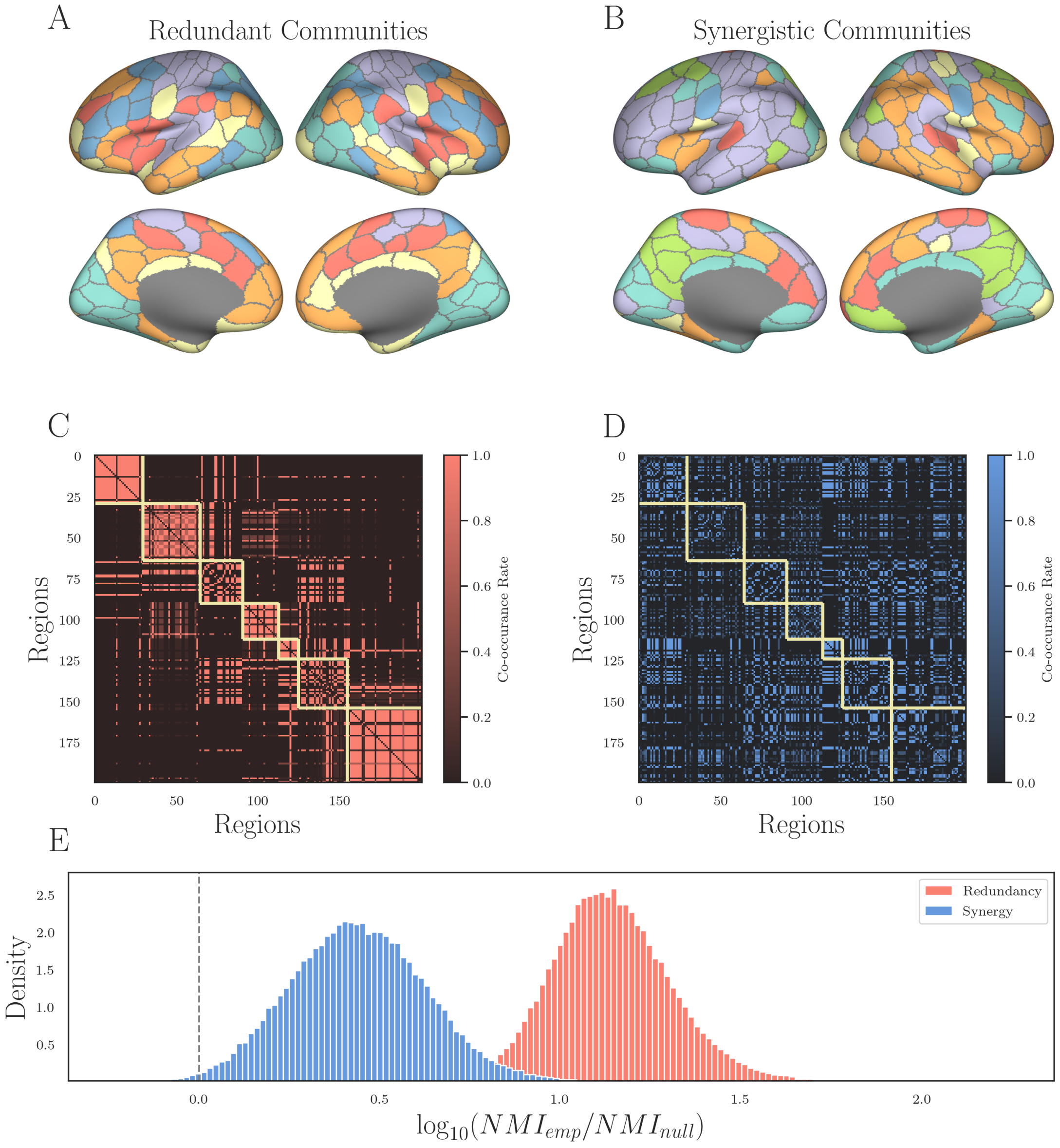
Poetto, Saggar, Battaglia, Vaccarino, Rabuffo, Petri in prep

Scaffold fingerprinting

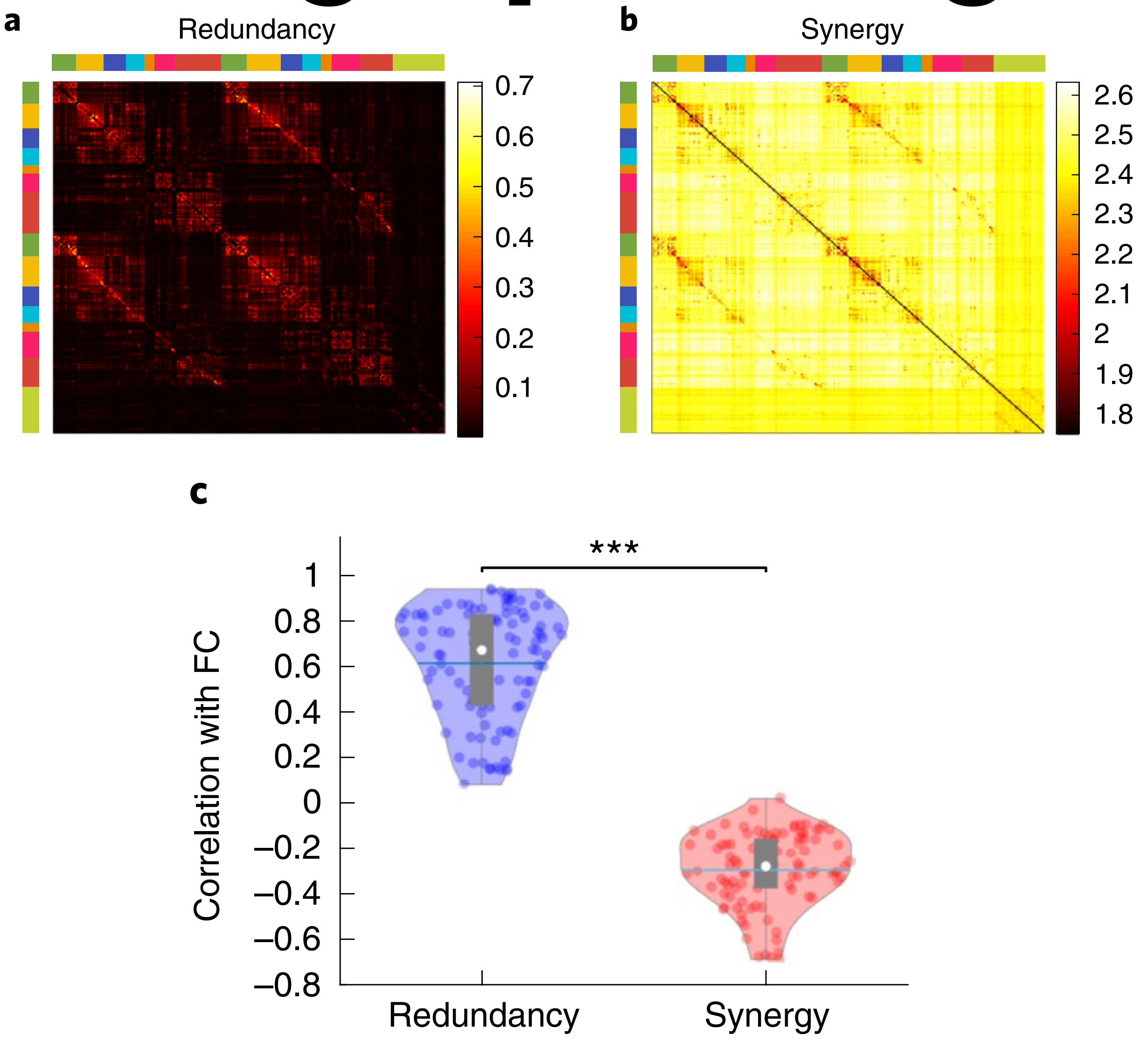
Summing up

- Consistently better
- Works for the short windows
- Sparse representation
- Ok, but why?

Brain informational fingerprinting

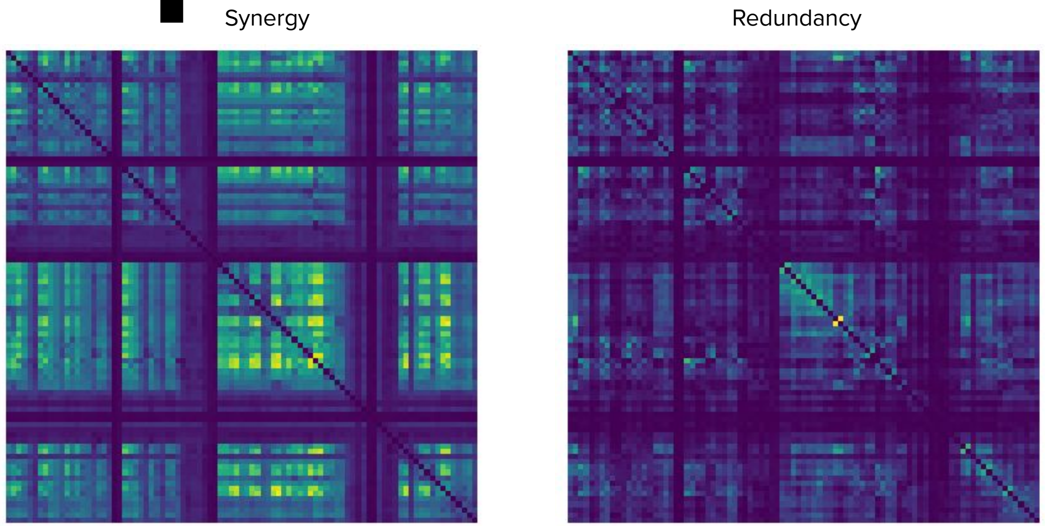


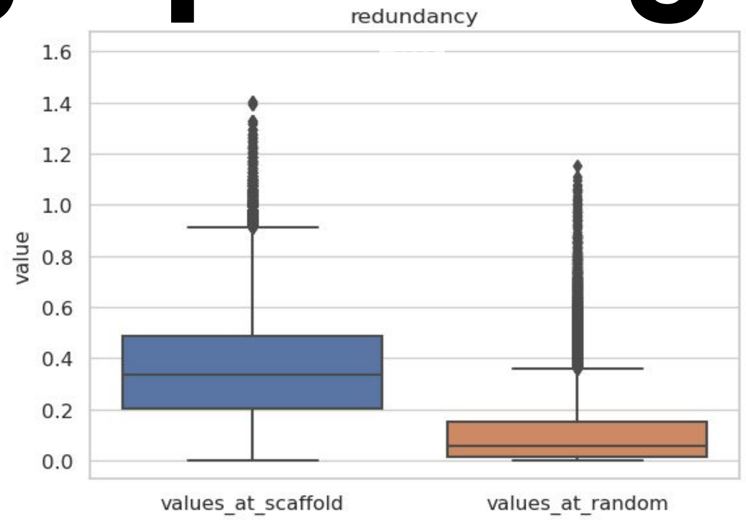
Varley, Thomas F., et al. *arXiv preprint arXiv:2301.05307* (2023).

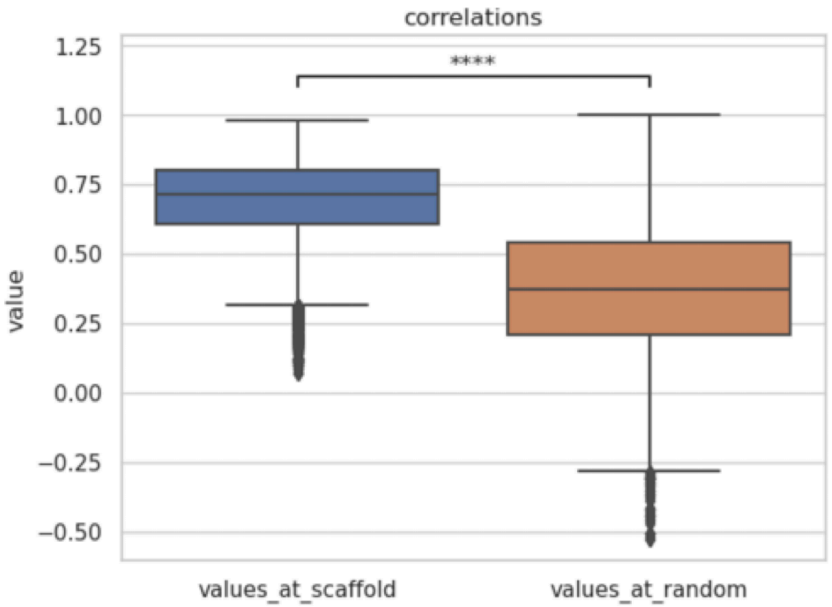


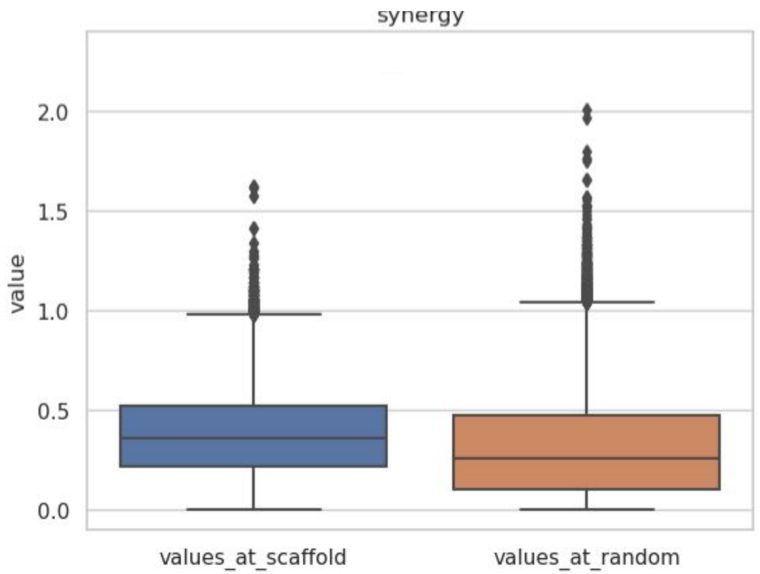
Luppi, Andrea I., et al. *Nature Neuroscience* 25.6 (2022): 771-782.

Topo+Info brain fingerprinting









Poetto, Saggar, Battaglia, Vaccarino, Rabuffo, Petri in prep

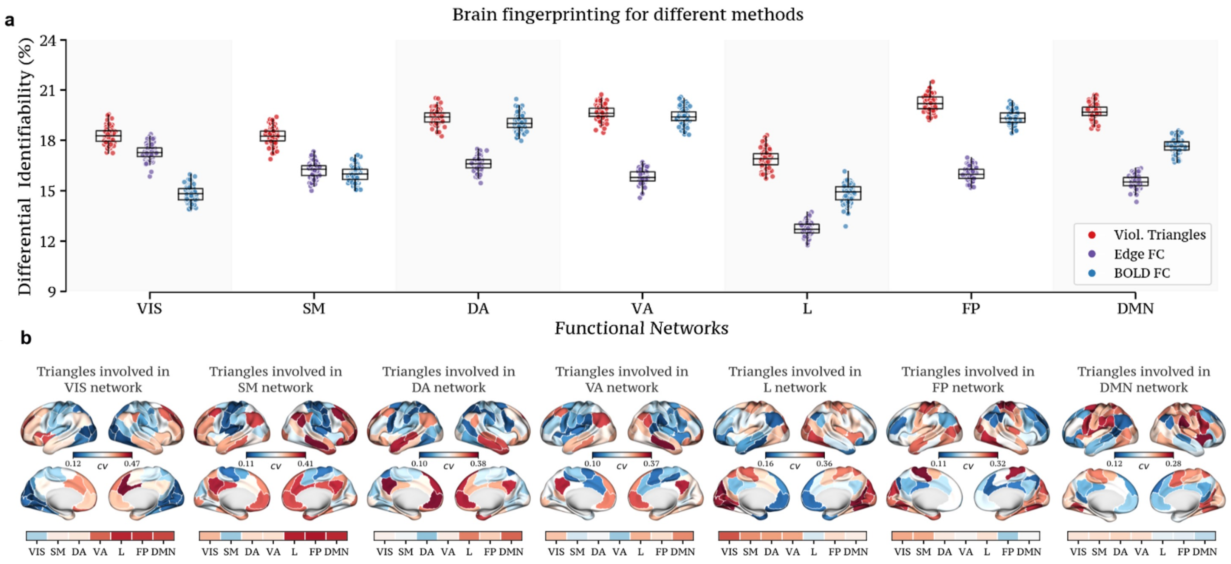
Topo+Info brain fingerprinting

Summing up

- Topological information (simplification) discriminates well across individuals
- Stronger effect for higher-order timeseries
- Global markers (Mapper) powerful
    - no relation to the actual synergy/ redundancy patterns
- Local markers (scaffold) even more powerful.
    - Related to local HOI info-theory, but not sufficient to explain

To do

- Time-resolved (a la Santoro, Andrea, et al. Nat. Phys. (2023))
- Distinguish by functional subnetwork
- Generative models of target topology



Santoro, Petri, Battiston, Amico, out soon!

Talk to me @lordgrilo Check stuff out @ lordgrilo.github.io

NPL RESEARCH

CETI THEORY

Network Science Institute
at Northeastern University
We are hiring Phds+postdocs (in London!)

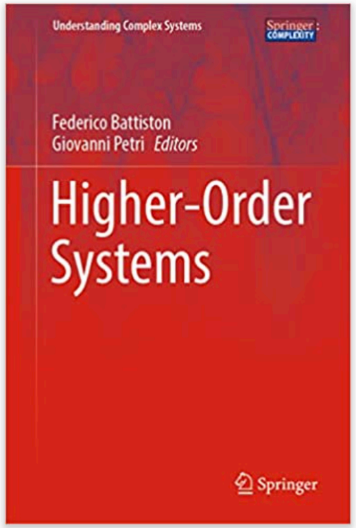
nature physics

PERSPECTIVE
https://doi.org/10.1038/s41567-021-01371-4

Check for updates

The physics of higher-order interactions in complex systems

Federico Battiston[1]✉, Enrico Amico[2,3], Alain Barrat[4,5], Ginestra Bianconi[6,7], Guilherme Ferraz de Arruda[8], Benedetta Franceschiello[9,10], Iacopo Iacopini[1], Sonia Kéfi[11,12], Vito Latora[6,13,14,15], Yamir Moreno[8,15,16,17], Micah M. Murray[9,10,18], Tiago P. Peixoto[1,19], Francesco Vaccarino[20] and Giovanni Petri[8,21]✉



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Contributors: Bianconi, Krioukov, Moreno, Barrat, Scarpino, Jost, Vaccarino, Bobrowski, Arenas, Skardal, Bick, Porter, Pikowski, Lambiotte, Schaub, ...

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Manish Saggar


Simone Poetto


Francesco Vaccarino


Demian Battaglia


Giovanni Rabuffo


Thanks!

Slides here: