Locating fast-varying line disturbances with the frequency mismatch

Robin Delabays,[1] Laurent Pagnier,[2] and Melvyn Tyloo[3]

[1] Center for Control, Dynamical Systems, and Computation, UC Santa Barbara, USA.
[2] Department of Mathematics, University of Arizona, Tucson, USA.
[3] Theoretical Division, Los Alamos National Laboratory, USA.

robindelabays@ucsb.edu, laurentpagnier@math.arizona.edu, mtyloo@lanl.gov

UC SANTA BARBARA

THE UNIVERSITY OF ARIZONA

Los Alamos NATIONAL LABORATORY

Line disturbances and the frequency mismatch

In diffusive networks of dynamical agents,

$$m_i\ddot{x}_i + d_i\dot{x}_i = \omega_i - \sum_{j=1}^{n} a_{ij} f(x_i - x_j)\,, \qquad i \in \{1,...,n\}\,, \tag{1}$$

we can have **nodal disturbances** which are **additive**, and **line distrubances** which are **multiplicative**,

$$\boldsymbol{\omega}(t) = \boldsymbol{\omega}^* + \xi_{\rm n}(t)\boldsymbol{e}_i\,, \qquad A(t) = A^* + \xi_{\rm l}(t)\boldsymbol{e}_{ij}\boldsymbol{e}_{ij}^\top\,, \tag{2}$$

with $\boldsymbol{e}_i = (0,...,1,...,0)^\top$ and $\boldsymbol{e}_{ij} = \boldsymbol{e}_i - \boldsymbol{e}_j$.

> Locating lines disturbances from time series is hard in general.
>
> We propose to use the **frequency mismatch**
>
> $$\boldsymbol{\psi}(t) = L_f(0)\boldsymbol{x}(t)\,. \tag{3}$$

Slow disturbances [1]

At steady state, $\boldsymbol{x} \approx L_f^\dagger\boldsymbol{\omega}$ and if the disturbance is sufficiently slow,

$$\boldsymbol{x}(t) \approx [L_f(t)]^\dagger\boldsymbol{\omega}\,. \tag{4}$$

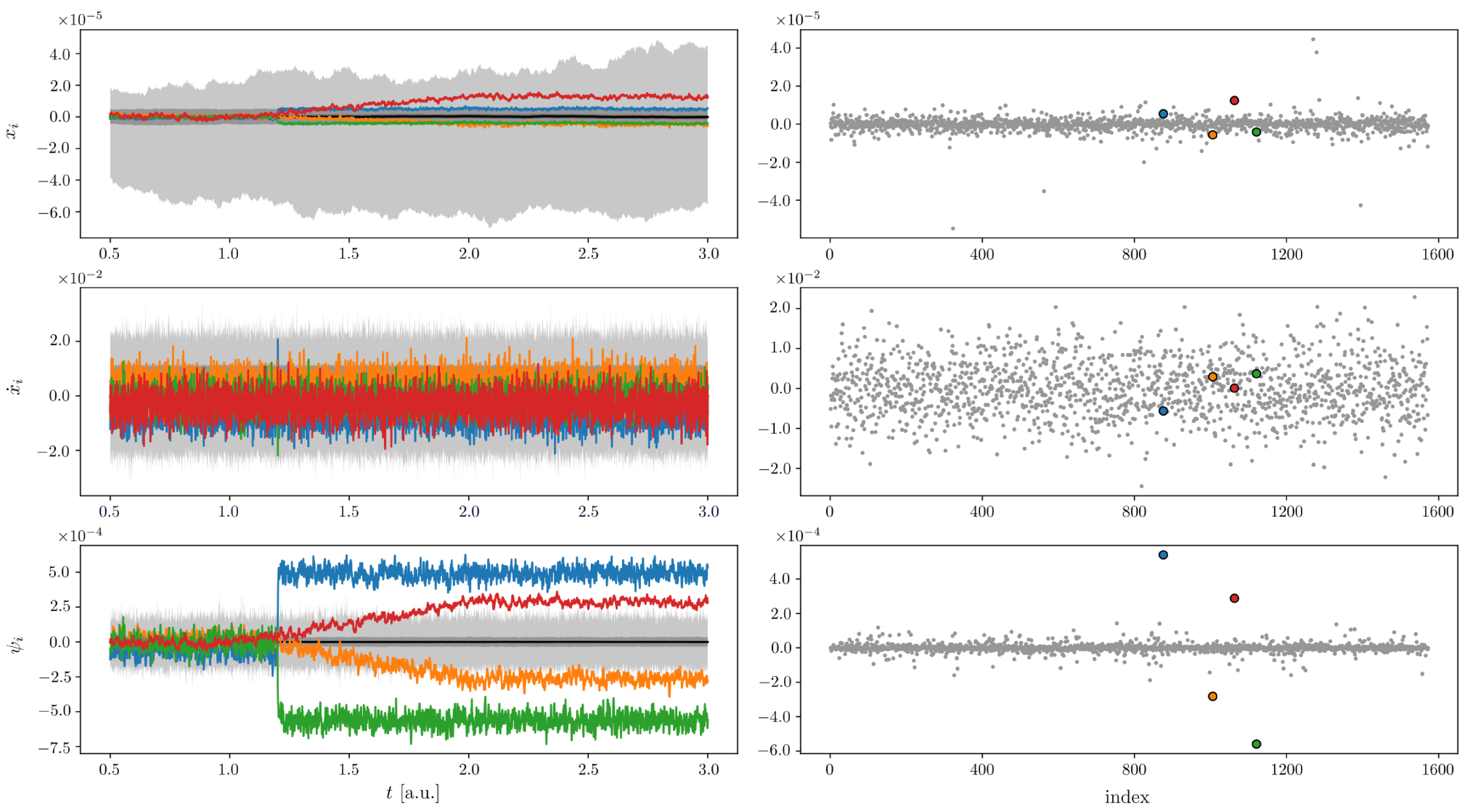
Using the Sherman-Morrison formula yields

$$\boldsymbol{\psi}(t) = \boldsymbol{\omega} - \alpha(t)\left(\boldsymbol{e}_{ij}^\top L_f^\dagger\boldsymbol{\omega}\right)\boldsymbol{e}_{ij}\,, \qquad \alpha(t) = \frac{\xi_{\rm l}(t)}{1+\xi_{\rm l}(t)\boldsymbol{e}_{ij}^\top L_f^\dagger\boldsymbol{e}_{ij}}\,, \tag{5}$$

which pin-points the ends of the disturbed line as the two nodes with largest amplitude

$$\eta_i = \max_{t\geq 0}\psi_i(t) - \min_{t\geq 0}\psi_i(t)\,. \tag{6}$$

Remark. *The same results can be obtained for partial measurements, using the Kron reduction of the systems [1].*



Positions, velocities, and frequency mismatch in the third order Kuramoto model [2] on the US airports network [3].

Confidence estimates

Let us define two node indexings, $\{i_1,...,i_n\}$ and $\{j_1,...,j_n\}$, such that

$$\eta_{i_1} \geq \cdots \geq \eta_{i_n}\,, \qquad \text{and} \qquad \eta'_{j_1} \geq \cdots \geq \eta'_{j_n}\,. \tag{7}$$

This allows us to define a *confidence level*

$$c_{\boldsymbol{\psi}} = 1 - \eta_{i_3}/\eta_{i_2}\,, \qquad c_{\boldsymbol{x}} = 1 - \eta'_{j_3}/\eta'_{j_2}\,. \tag{8}$$

Fast disturbances [4]

Sufficiently fast perturbations are approximated as a sum of *Dirac-deltas*,

$$\xi_{\rm l}(t) = \sum_k \xi_k\delta(t-k\tau)\,, \tag{9}$$

and at very short time scales, $t \ll 1$,

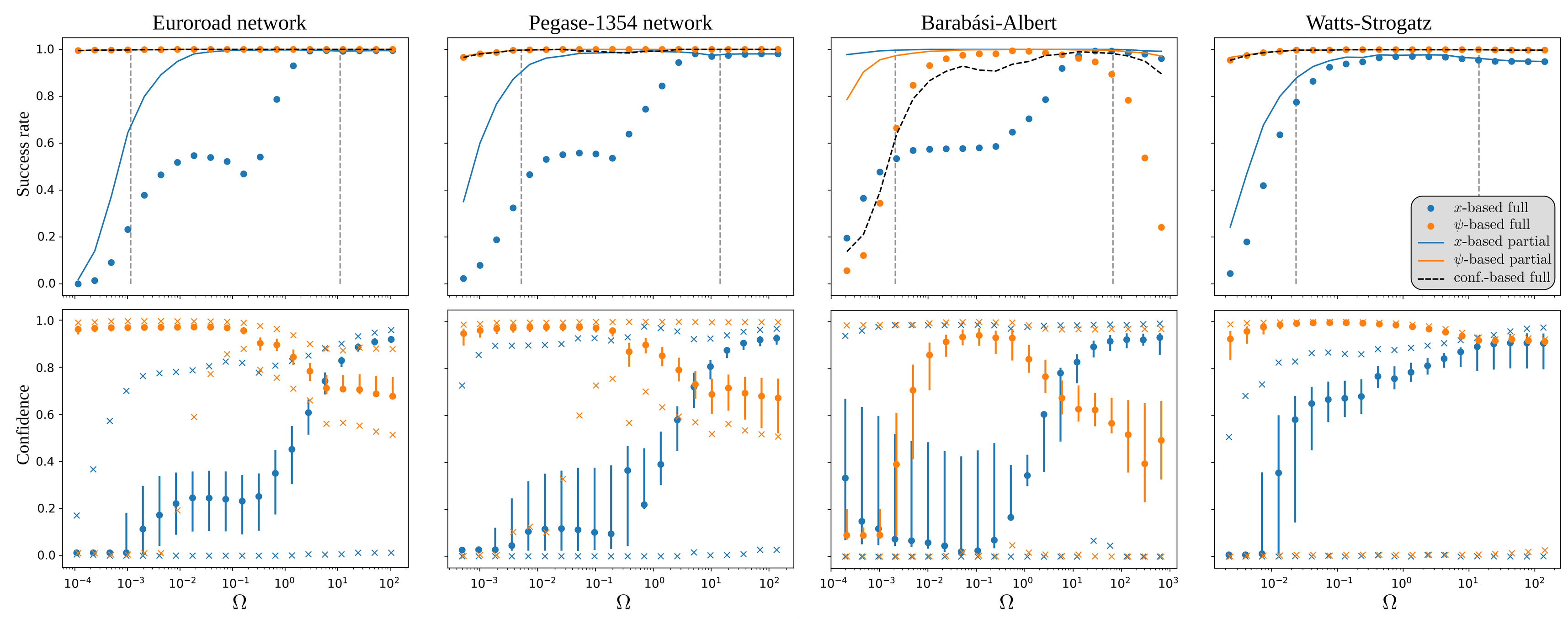
$$\boldsymbol{x}(t) \approx \boldsymbol{x}^* + \xi_0(x_i^* - x_j^*)\boldsymbol{e}_{ij} + O(t)\,. \tag{10}$$

Therefore,

$$\boldsymbol{x}(t) \approx \boldsymbol{x}^* + \alpha'(t)(\boldsymbol{e}_i - \boldsymbol{e}_j)\,, \qquad \alpha'(t) = \sum_{k:\,k\tau<t}\xi_k\,, \tag{11}$$

which allows to identify the disturbed line, again as the link between the two nodes with largest amplitude

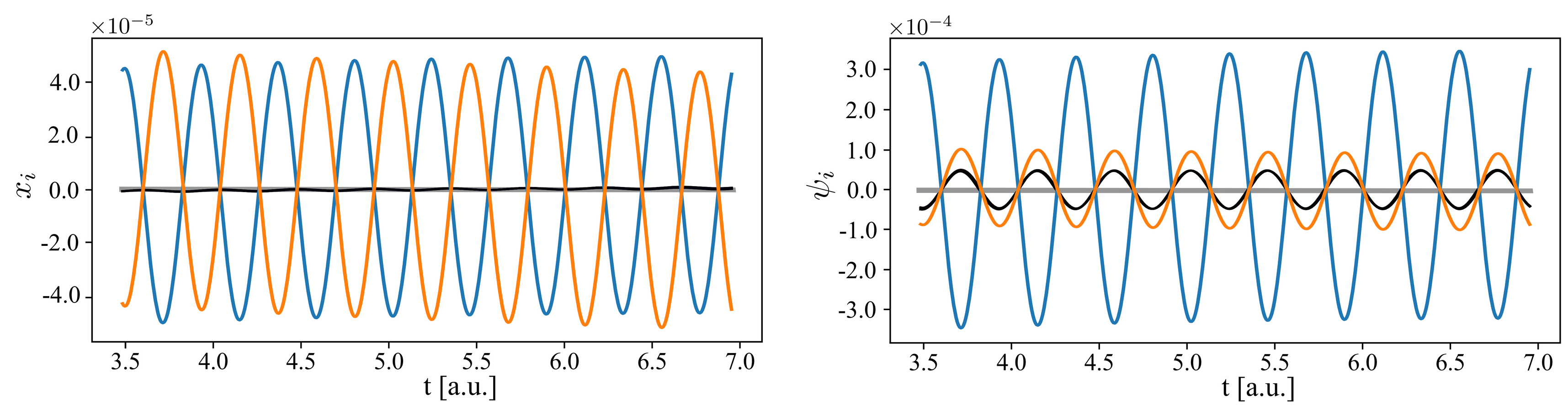
$$\eta'_i = \max_{t\geq 0}x_i(t) - \min_{t\geq 0}x_i(t)\,. \tag{12}$$



Success rate (top row) and confidence (bottom row) of the x-based (blue) and the ψ-based (orange) method, for four different network structures (columns).

The frequency mismatch works surprisingly well,

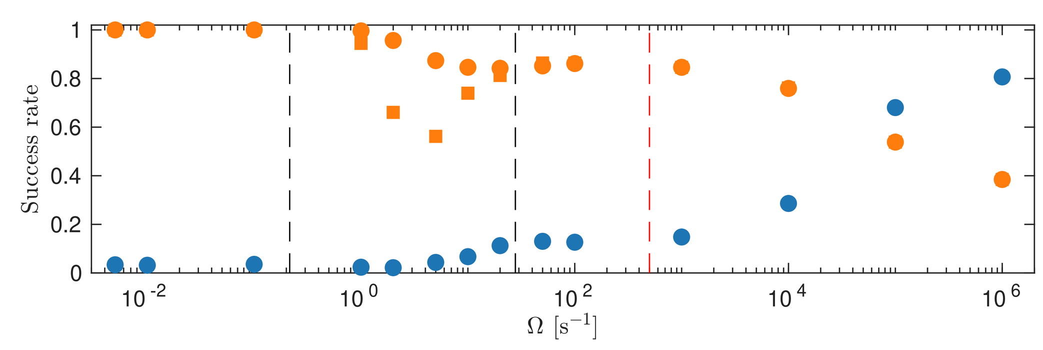
$$\psi_k(t) = \alpha'(t)\left(L_f\boldsymbol{e}_i - L_f\boldsymbol{e}_j\right)_k = \alpha'(t)\left(L_{ki} - L_{kj}\right) = \begin{cases}\alpha'(t)(\deg_i + a_{ij})\,, & \text{if } k=i\,,\\ \alpha'(t)(-a_{ij} - \deg_j\,, & \text{if } k=j\,,\\ \alpha'(t)(a_{kj} - a_{ki})\,, & \text{otherwise.}\end{cases} \tag{13}$$



Comparison of the times series of $\boldsymbol{x}$ and $\boldsymbol{\psi}$ for a fast disturbance.

PanTaGruEl

Application on the PanTaGruEl [5] model the European interconnected grid with second order dynamics and heterogeneous admittances.



References

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