UC SANTA BARBARA

Hes·so VALAIS WALLIS
School of Engineering

Complex Networks of Lossy Oscillators:
Multistability, Anomalies, and Loop Flows in Power Grids

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Acknowledgment

Saber Jafarpour (Georgia Tech) Francesco Bullo (UCSB)

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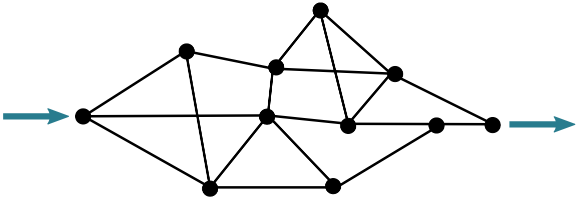
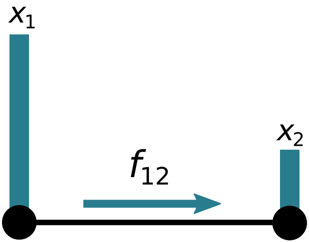
Swiss National
Science Foundation

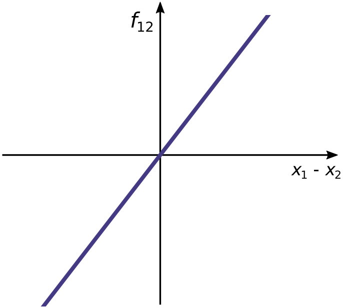
Flow networks

How is a commodity transmitted over a network?



Pairwise interaction





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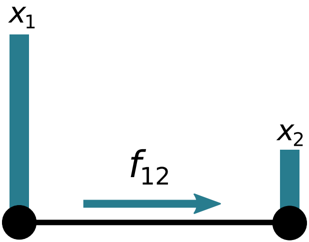
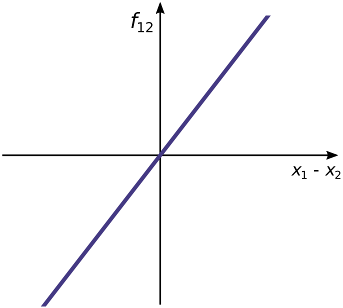




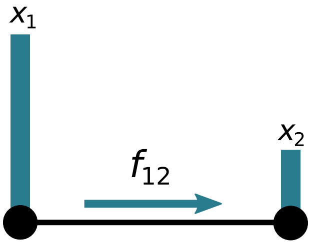
$a_{ij} = a_{ji}$

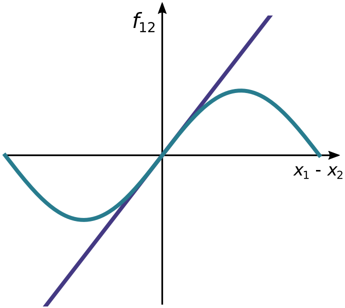
$$f_{ij} = a_{ij}(x_i - x_j)$$
$$f_{ji} = -f_{ij}$$

"DC approximation" of AC power flows:

$$P_{ij} = B_{ij}(\theta_i - \theta_j)$$

Pairwise interaction





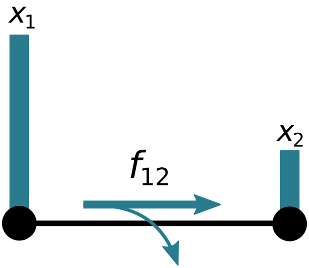
$a_{ij} = a_{ji}$ **and** $h(x) = -h(-x)$

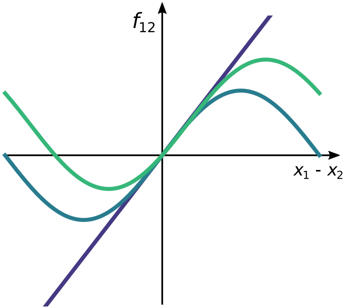
$$f_{ij} = a_{ij} h(x_i - x_j)$$
$$f_{ji} = -f_{ij}$$

Lossless approx. of AC power flows:

$$P_{ij} = B_{ij} \sin(\theta_i - \theta_j)$$

Pairwise interaction





$a_{ij} \neq a_{ji}$ **or** $h(x) \neq -h(-x)$

$$f_{ij} = a_{ij} h(x_i - x_j)$$
$$f_{ji} = a_{ji} h(x_j - x_i)$$

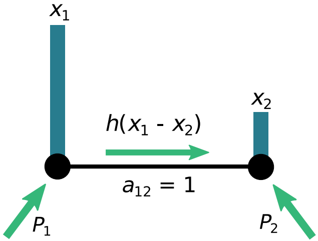
Active power flows:

$$P_{ij} = B_{ij} \sin(\theta_i - \theta_j - \phi)$$

Diffusive network - summary

$$\dot{x}_i = P_i - \sum_j a_{ij} h(x_i - x_j)$$

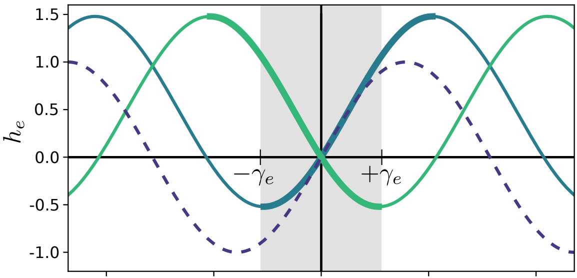
Potentially: $a_{ij} \neq a_{ji}$,
$h(x) \neq -h(-x)$.



- P_i: Natural frequency, commodity injection,...
- a_{ij}: Element of the adjacency matrix;
- h: Coupling function, flow function,...
- x_i: Agent's state.

The Kuramoto-Sakaguchi model

$$\dot{\theta}_j = P_j - \sum_k a_{jk} \sin(\theta_j - \theta_k - \phi)$$



H. Sakaguchi, S. Shinomoto, and Y. Kuramoto, *Local and Global Self-Entrainments in Oscillator Lattices*, Prog. Theor. Phys., **77**(5):1005–1010 (1987). DOI: 10.1143/PTP.77.1005

Dynamics on the n-torus

Phase oscillators: 2π-periodic coupling.

$$\dot{x}_i = P_i - \sum_j a_{ij} \sin(x_i - x_j)$$

From Euclidean space to the torus:

$$x_i \in \mathbb{R} \quad \rightarrow \quad \theta_i \in \mathbb{S}^1 = [-\pi, \pi)$$

$$x \in \mathbb{R}^n \quad \rightarrow \quad \theta \in (\mathbb{S}^1)^n = \mathbb{T}^n$$

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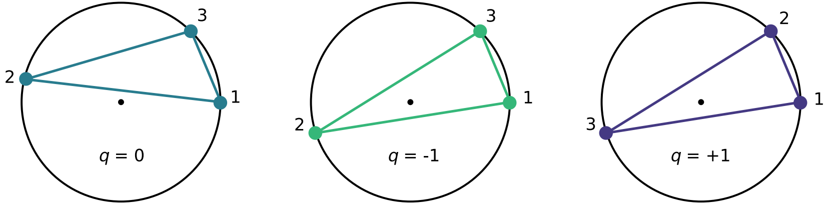
Source: Shutterstock.

$\to$

Source: Krispi Kreme ad.

Winding number and loop flows

Given a cycle $\sigma = (i_1, ..., i_\ell)$: $\quad q = (2\pi)^{-1} \sum_{k=1}^{\ell} d_{cc}(\theta_{i_k}, \theta_{i_{k-1}}) \in \mathbb{Z}.$



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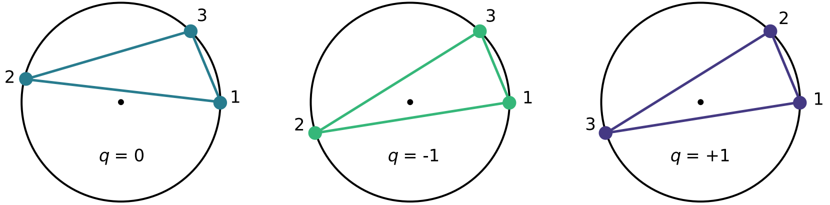
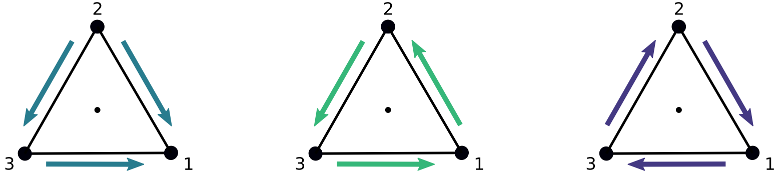


Loop flows:



Winding vectors and partition

For a cycle σ.
The **winding number**:

$$\begin{aligned} q_\sigma : \mathbb{T}^n &\to \mathbb{Z} \\ \theta &\mapsto q_\sigma(\theta) \end{aligned}$$

For a cycle basis $\Sigma = (\sigma_1, ..., \sigma_c)$.
The **winding vector**:

$$\begin{aligned} q_\Sigma : \mathbb{T}^n &\to \mathbb{Z}^c \\ \theta &\mapsto [q_{\sigma_1}(\theta), ..., q_{\sigma_c}(\theta)] \end{aligned}$$

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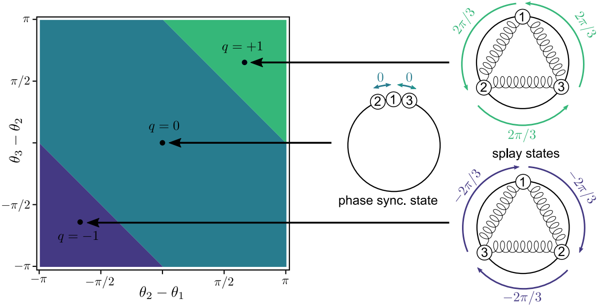
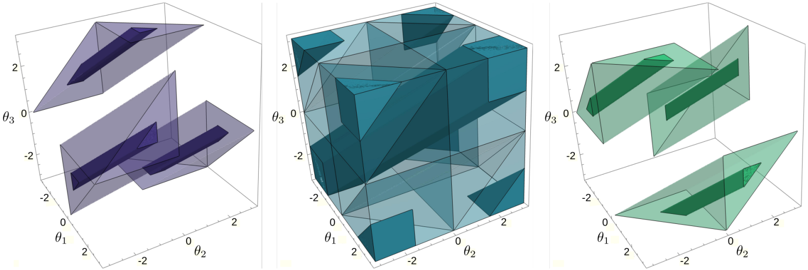


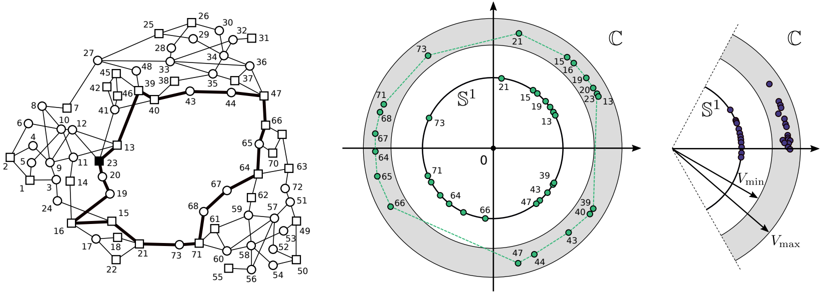
At most uniqueness within winding cells



S. Jafarpour, E. Y. Huang, K. D. Smith, and F. Bullo, *Flow and Elastic Networks on the n-Torus: Geometry, Analysis, and Computation*, SIAM Review **64** (2022). DOI: 10.1137/18M1242056

RD, S. Jafarpour, and F. Bullo, *Multistability and anomalies in oscillator models of lossy power grids*, Nat. Commun. **13**, 5238 (2022). DOI: 10.1038/s41467-022-32931-8

Loop flows in power grids?



Thank you!

RD, S. Jafarpour, and F. Bullo, *Multistability and anomalies in oscillator models of lossy power grids*, Nat. Commun. **13**, 5238 (2022).

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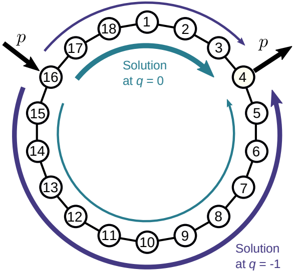
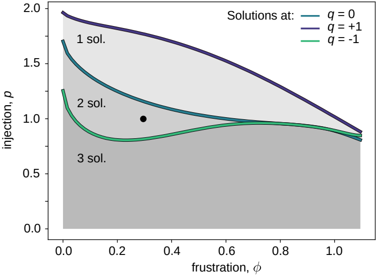
Anomaly 1: "Loop flows increase capacity."

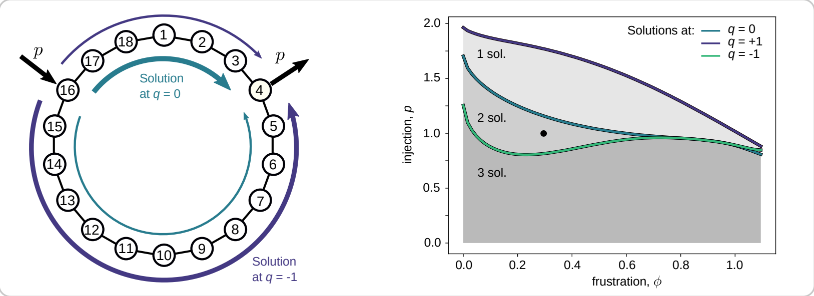
$$\dot{\theta}_i = P_i - \sum_j a_{ij} \sin(\theta_i - \theta_j - \phi)$$





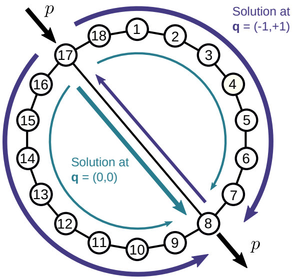
Anomaly 2: "Frustration increases capacity."

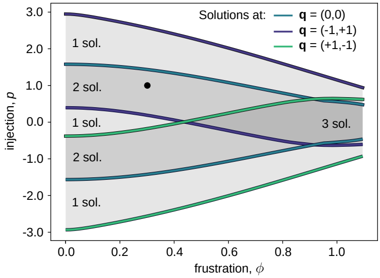
$$\dot{\theta}_i = P_i - \sum_j a_{ij} \sin(\theta_i - \theta_j - \phi)$$



Anomaly 3: "Frustration promotes multistability."

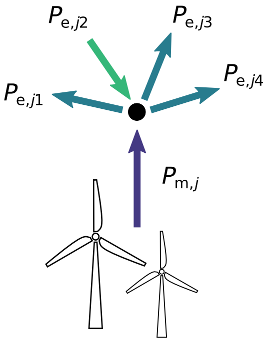
$$\dot{\theta}_i = P_i - \sum_j a_{ij} \sin(\theta_i - \theta_j - \phi)$$





The power flow equations

- Voltage: $V_j e^{i\theta_j}$.
- Power: $P_j + iQ_j$.
- Admittance: $G_{jk} + iB_{jk}$.
- Electrical power flow: $P_{\mathrm{e},jk}$.



J. Machowski, J. W. Bialek, and J. R. Bumby, *Power System Dynamics*, 2nd ed. (Wiley, Chichester, U.K, 2008).

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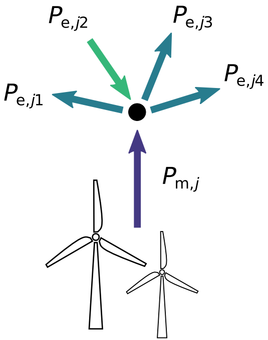

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- Electrical power flow: $P_{\mathrm{e},jk}$.

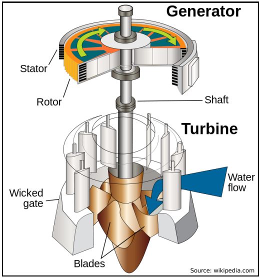
$$P_j = \sum_k V_j V_k \left[B_{jk} \sin(\theta_j - \theta_k) + G_{jk} \cos(\theta_j - \theta_k)\right] ,$$

$$Q_j = \sum_k V_j V_k \left[G_{jk} \sin(\theta_j - \theta_k) - B_{jk} \cos(\theta_j - \theta_k)\right] .$$

J. Machowski, J. W. Bialek, and J. R. Bumby, *Power System Dynamics*, 2nd ed. (Wiley, Chichester, U.K, 2008).

The swing equations

$$m_j\ddot{\theta}_j + d_j\dot{\theta}_j = P_{\mathrm{m},j} - P_{\mathrm{e},j}$$



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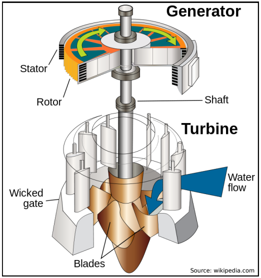

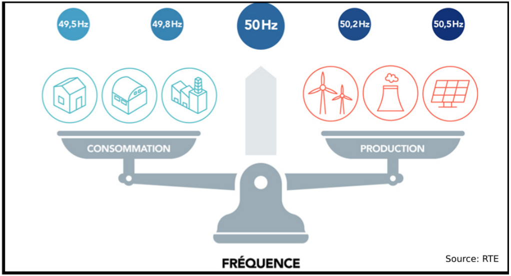

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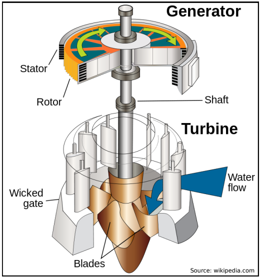
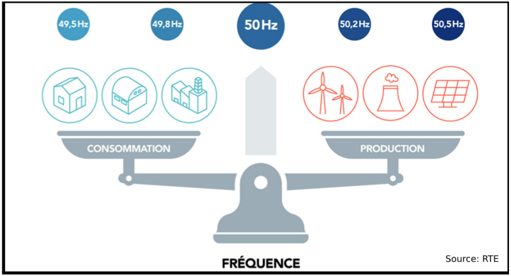
The swing equations

$$
\begin{aligned}
m_j\ddot{\theta}_j + d_j\dot{\theta}_j &= P_{\mathrm{m},j} - P_{\mathrm{e},j} \\
&= P_j - \sum_k B_{jk}\sin(\theta_j - \theta_k) + G_{jk}\cos(\theta_j - \theta_k)
\end{aligned}
$$





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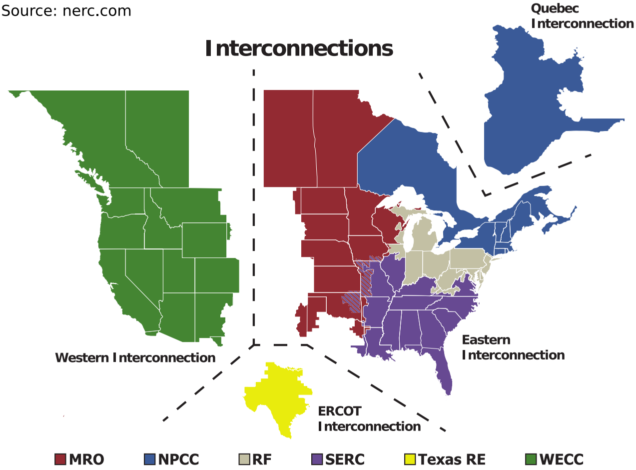
Synchronous power grid

Source: nerc.com



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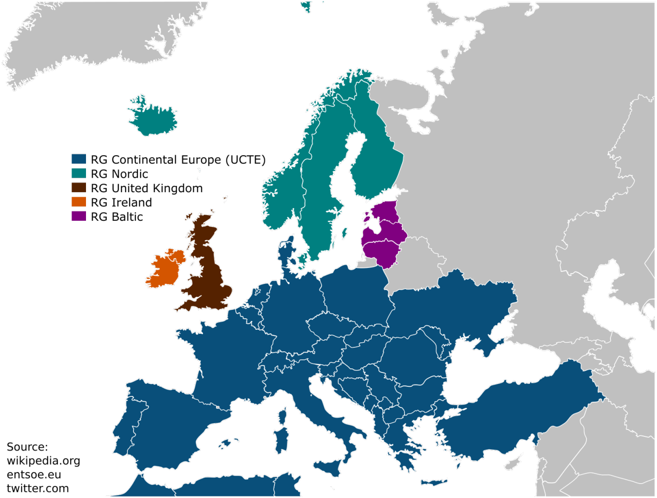
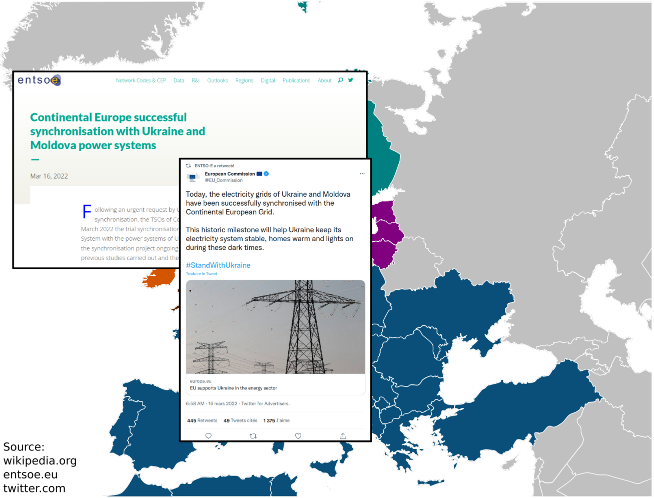


Source:
wikipedia.org
entsoe.eu
twitter.com

Synchronous power grid



Source:
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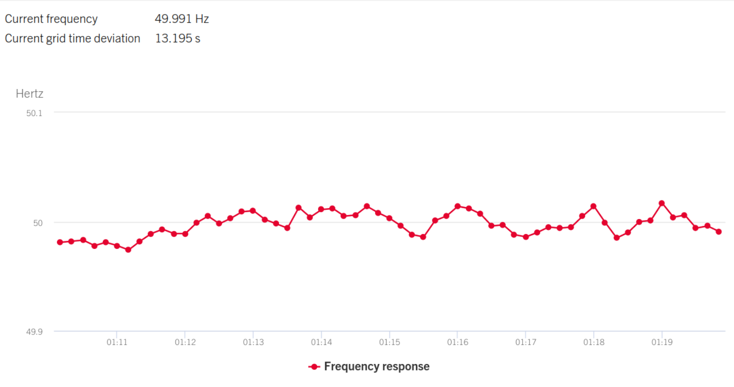
Grid frequency



Source: www.swissgrid.ch (Apr. 26, 2022)