UC SANTA BARBARA

Reconstructing Network Structures from Partial Measurements

Robin Delabays

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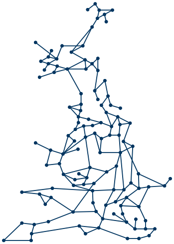
Joint work with P. Jacquod and M. Tyloo

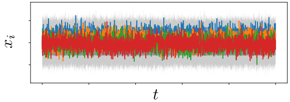
Interplay between networks and dynamics



Influence

Inference

$$\dot{x}_i = \sum_j F_{ij}(x_i, x_j) + \xi$$



W.-X. Wang, Y.-C. Lai, and C. Grebogi, *Phys. Rep.* **644** (2016).

I. Brugere, B. Gallagher, and T. Y. Berger-Wolf, *ACM Comput. Surv.* **51** (2018).

PNAS

Revealing strengths and weaknesses of methods for gene network inference

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Edited by Charles R Cantor, Sequenom Inc., S…

Numerous methods have been develop… latory networks from expression data, … lute and comparative performance rem… this paper, we introduce a framework f… sessment of methods for gene network i… *silico* benchmark suite that we provided… wide challenge within the context of th… verse Engineering Assessment and Meth… performance of 29 gene-network-infere… been applied independently by particip… profiling reveals that current inference… various degrees, by different types of sy… In particular, all but the best-performin…

SCIENCE ADVANCES | RESEARCH ARTICLE

APPLIED SCIENCES AND ENGINEERING

The key player problem in complex oscillator networks and electric power grids: Resistance centralities identify local vulnerabilities

M. Tyloo[1,2], L. Pagnier[1,2], P. Jacquod[2,3]*

Identifying key players in coupled individual systems is a fundamental problem in network theory. We investigate … coupled oscil… …gest excursion … to this identi… …the sense of a …anking is effi… …ous state. The …atrix inversion …nonlinearities,

PNAS

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SEE COMMENTARY

Fundamental structures of dynamic social networks

Vedran Sekara[a], Arkadiusz Stopczynski[a,b], and Sune Lehmann[a,c,1]

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Social systems are in a constant state of flux, with dynamics spanning from minute-by-minute changes to patterns present on the timescale of years. Accurate models of social dynamics are important for understanding the spreading of influence or diseases, formation of friendships, and the productivity of teams. Although there has been much progress on understanding complex networks over the past decade, little is known about the regularities governing the micro-dynamics of social networks. Here, we explore the dynamic social network of a densely-connected population of ∼1,000 individuals and their interactions in the network of real-world person-to-person proximity measured via Bluetooth, as well as their telecommunica…

interactions in the network of physical proximity measured via Bluetooth (*Materials and Methods*), complemented with information from telecommunication networks (phone calls and text messages), online social media (Facebook interactions), as well as geolocation and demographic data.

Until this point, community detection in dynamic networks has required complex mathematical heuristics (14, 15). Here, we show that with high-resolution data describing social interactions, community detection is unnecessary. When single time slices are shorter than the rate at which social gatherings change, communities of individuals can be observed directly and with little ambiguity (Fig.

Various approaches

Probing: [Yu et al., *Phys. Rev. Lett.* **97** (2006)], [Timme, *Phys. Rev. Lett.* **98** (2007)], [Dong et al., *PLoS ONE* **8** (2013)], [Basiri et al., *Phys. Rev. E* **98** (2018)], [Tyloo and D., *J. Phys. Complexity* **2** (2021)], ...

Maximum likelihood/cost minimization: [Hoang et al., *Phys. Rev. E* **99** (2019)], [Makarov et al., *J. Neurosci. Methods* **144**(2005)], [Shandilya and Timme, *New J. Phys.* **13** (2011)], [Panaggio et al., *Chaos* **29** (2019)], ...

Statistical properties of trajectories: [Dahlhaus et al., *J. Neurosci. Methods* **77** (1997)], [Sameshima and Baccalá, *J. Neurosci. Methods* **94** (1999)], [Ren et al., *Phys. Rev. Lett.* **104** (2010)], [Newman, *Nature Physics* **14** (2018)], [Peixoto, *Phys. Rev. Lett.* **123** (2019)], ...

The model

Dynamics:

$$\dot{x}_i(t) = -F_i[x(t)] + \xi_i(t) \qquad\qquad i \in \{1, ..., n\}\,.$$

Network structure:

$$i \sim j \qquad\qquad \iff \qquad\qquad (\mathcal{J}_F)_{ij} = \frac{\partial F_i}{\partial x_j} \neq 0\,.$$

Linearization around x^*:

$$\delta = x - x^*\,, \qquad\qquad \dot{\delta} = -\mathcal{J}_F(x^*)\delta + \xi + O(\|\delta\|^2)\,.$$

Assumptions:

- $\mathcal{J}_F$ is symmetric;
- $\mathcal{J}_F$ is positive semidefinite.

Eigendecomposition of $\mathcal{J}$

Real eigenvalues: $0 \leq \lambda_1 \leq \ldots \leq \lambda_n$.

Orthogonal eigenvectors: $u_1, \ldots, u_n$.

$$\delta(t) = \sum_{i=1}^{n} c_i(t) u_i \,,$$

$$\dot{c}_i(t) = -\lambda_i c_i(t) + u_i^\top \xi \,,$$

$$c_i(t) = e^{-\lambda_i t} \int_0^t e^{\lambda_i t'} u_i^\top \xi(t') \mathrm{d}t' \,.$$

Two-point velocity correlator

Noise: **correlated in time, uncorrelated in space**:

$$\langle \xi_i(t) \rangle = 0\,, \qquad \langle \xi_i(s)\xi_j(t) \rangle = \xi_0^2 \delta_{ij} \exp(-|s-t|/\tau_0)\,.$$

$$\lim_{t\to\infty} \langle \dot{\delta}_i \dot{\delta}_j \rangle = \cdots$$

$$= \xi_0^2 \left[\delta_{ij} + \sum_{\ell \geq 1} (-\tau_0)^\ell (\mathcal{J}^\ell)_{ij} \right] \,.$$

M. Tyloo, R.D., and P. Jacquod, arXiv preprint:2007.16136 (2020).

Direct reconstruction

$$\tau_0 \ll 1 \qquad \Longrightarrow \qquad \mathcal{J}_{ij} \approx \left(\delta_{ij} - \langle \dot{\delta}_i \dot{\delta}_j \rangle / \xi_0^2 \right) / \tau_0 \,.$$

$$n = 3809$$

$$m = 4944 \,.$$

L. Pagnier and P. Jacquod, *PanTaGruEl*, Zenodo Repository (2019). doi.org/10.5281/zenodo.2642175

Direct reconstruction

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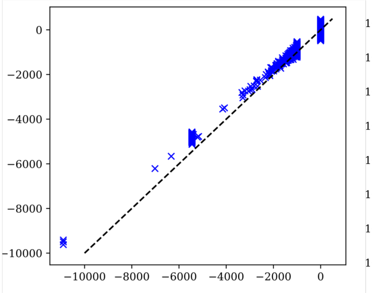
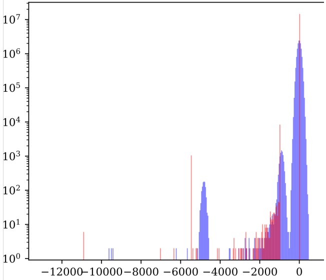


$\mathbb{J}_{ij}$



$\mathbb{J}_{ij}$

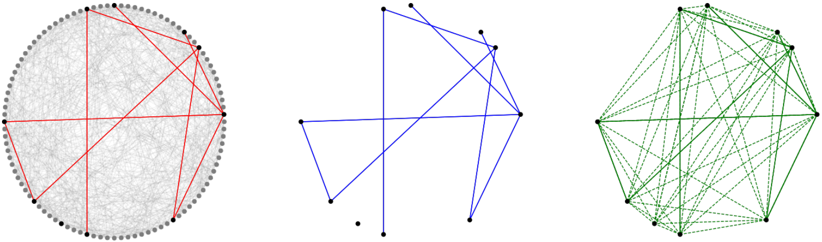
L. Pagnier and P. Jacquod, *PanTaGruEl*, Zenodo Repository (2019). doi.org/10.5281/zenodo.2642175

Partial measurements

Previous approaches infer $\mathcal{J}^{-1}$.

Our approach:

$$\mathcal{J}_{ij} = \left(\delta_{ij} - \langle\dot{\delta}_i\dot{\delta}_j\rangle/\xi_0^2\right)/\tau_0$$



Wrap-up

www.arxiv.org/abs/2007.16136

Costs (assumptions):

- Stable fixed point;
- Symmetric coupling;
- Short correlation time.

Benefits:

- Direct reconstruction;
- (Geodesic distance;)
- Partial measurements.

Thank you!

Conference on Complex Systems 2021 - Satellite Symposium
Data-based diagnosis of networked systems
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Geodesic distances

$$\ell < d_{ij} \implies (\mathcal{J}^{\ell})_{ij} = 0$$

$$\lim_{t\to\infty} \langle \dot{\delta}_i \dot{\delta}_j \rangle = \xi_0^2 \sum_{\ell=d_{ij}}^{\infty} (-\tau_0)^{\ell} (\mathcal{J}^{\ell})_{ij} \sim \tau_0^{d_{ij}} \, .$$