Robustness of elections results against external influence

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Motivation

Electrical networks:

- Dynamical systems:

$$m_i\ddot{\theta}_i + d_i\dot{\theta}_i = \omega_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)\,,$$

- Complex networks:

Where else can we apply our results?

The model

Taylor model (continuous time French-DeGroot):

$$\dot{x}_i = \sum_j w_{ij}(x_j - x_i) + \alpha_i(x_i^{(0)} + \omega_i - x_i)\,,$$

- x_i: i's opinion;
- w_{ij}: weight accorded to j by agent i;
- $x_i^{(0)}$: i's natural opinion;
- ω_i: external influence on i;
- α_i: attachment of i to their natural opinion.

Consider $w_{ij} = a_{ij}$, $\alpha_i = 1$, and $\omega_i \in \{-1, 0, 1\}$.

M. Taylor, *Human Relations* **21** (1968).

The model

$$\dot{\mathbf{x}} = -(\mathbb{L} + \mathbb{I})\mathbf{x} + \mathbf{x}^0 + \boldsymbol{\omega}\,,$$

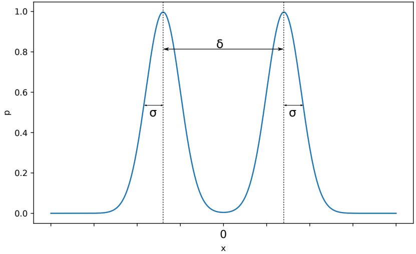
$$\Longrightarrow \mathbf{x}^* = (\mathbb{L} + \mathbb{I})^{-1}(\mathbf{x}^0 + \boldsymbol{\omega})$$

Outcome:

$$o(\mathbf{x}) = \sum_i \mathrm{sign}(x_i)$$

Assumed positive (wlog) before influence.

Natural opinions

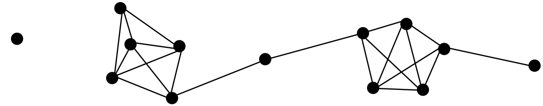


- Bimodal $\implies$ models polarization;
- Symmetric $\implies$ close to parity;
- Random $\implies$ avoids degeneracies.

The network

$$(\mathbb{L}_\epsilon)_{ij} = \begin{cases} -1\,, & i \neq j, |x_i^{(0)} - x_j^{(0)}| \leq \epsilon\,, \\ 0\,, & i \neq j, |x_i^{(0)} - x_j^{(0)}| > \epsilon\,, \\ -\sum_{k\neq j} (\mathbb{L}_\epsilon)_{ik}\,, & i = j\,. \end{cases}$$

ϵ



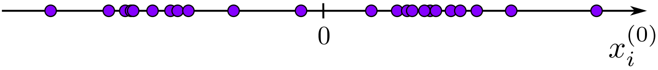
Influence strategies

Goal: Change the outcome by influencing the agents, minimizing the effort, i.e., $\|\boldsymbol{\omega}\|_1$.

Question: Who should we target?

Proposed strategies:

- Random
- Minimum



- Fiedler ...

Fiedler strategy

Maximize the impact of $\boldsymbol{\omega}$ on the outcome, "i.e.", on $\mathbf{x}^*$.

$$\boldsymbol{\delta}\mathbf{x} := \mathbf{x}^*(\boldsymbol{\omega}) - \mathbf{x}^*(\mathbf{0}) = (\mathbb{L} + \mathbb{I})^{-1} (\mathbf{x}^0 + \boldsymbol{\omega} - \mathbf{x}^0)$$

Decompose on the Laplacian's eigenbasis:

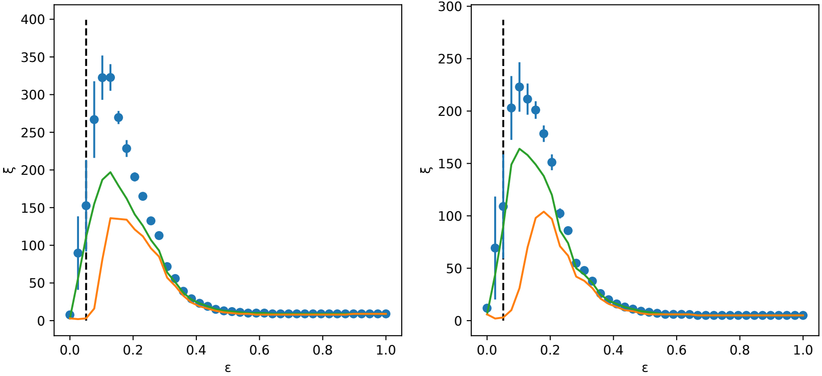
$$0 = \lambda_1 \leq \lambda_2 \leq ... \leq \lambda_n \,, \qquad \mathbf{u}^{(1)} = n^{-1/2}(1, ..., 1)\,,$$

$$\boldsymbol{\delta}\mathbf{x} = \sum_i \frac{\boldsymbol{\omega}^\top \mathbf{u}^{(i)}}{\lambda_i + 1} \mathbf{u}^{(i)}\,.$$

$\Longrightarrow$ try to align $\boldsymbol{\omega}$ with the *slow modes* of $\mathbb{L}$ (but not $\mathbf{u}^{(1)}$),
$\Longleftrightarrow$ influence agents with large *resistive centrality*.

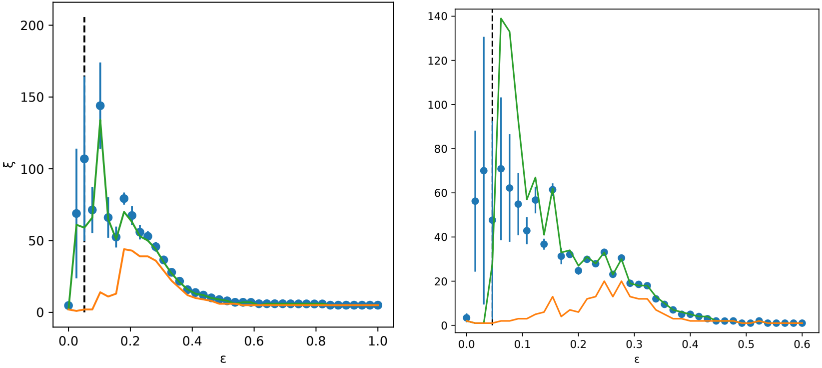
Results

Random, Minimum, Fiedler



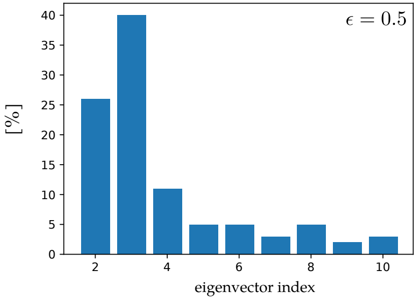
Results

Random, Minimum, Fiedler



Results

Random, Minimum, Fiedler



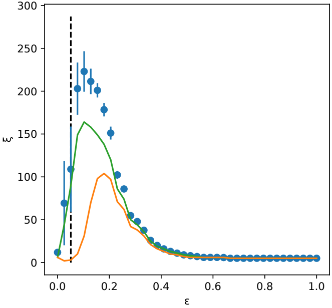
Further questions (and conclusion)

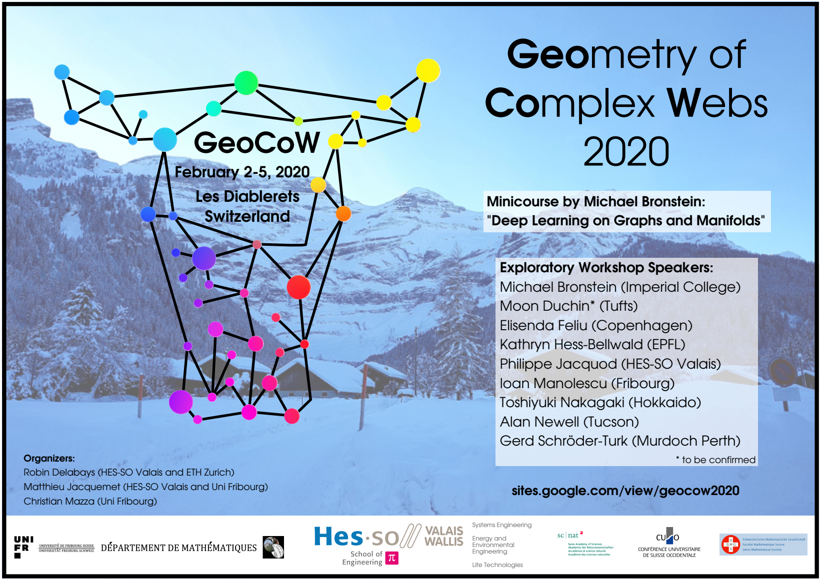
To do:

- Desintricate the "Fiedler strategy";
- Determine the role of parameters:

$$\dot{x}_i = \sum_j w_{ij}(x_j - x_i) + \alpha_i(x_i^{(0)} + \omega_i - x_i)\,.$$

Observation: Intermediate interaction implies the most robust opinions.

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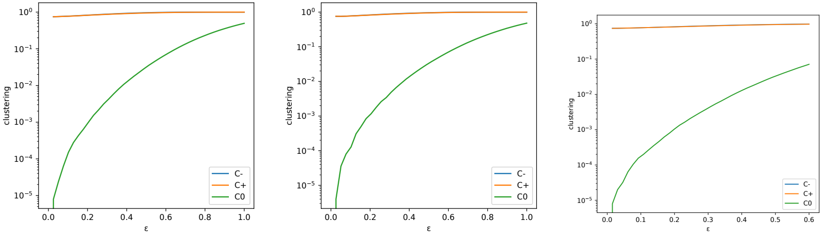
Clusterings



Resistive centrality

Resistance distance:

$$\Omega_{ij} = \mathbb{L}_{ii} + \mathbb{L}_{jj} - \mathbb{L}_{ij} - \mathbb{L}_{ji} \,.$$

Centrality:

$$C(k) = \left[n^{-1} \sum_j \Omega_{kj} \right]^{-1} = \left[\sum_{\ell \geq 2} \frac{(u_k^\ell)^2}{\lambda_\ell} + n^{-2} K f_1 \right]^{-1}$$

Kirchhoff index:

$$K f_1 = \sum_{i<j} \Omega_{ij} = n \sum_{\ell \geq 2} \lambda_\ell^{-1} \,.$$

M.Tyloo and P. Jacquod, *Phys. Rev. E* **100** (2019).