Bounding the desynchronization time in electrical grids under fluctuating sources

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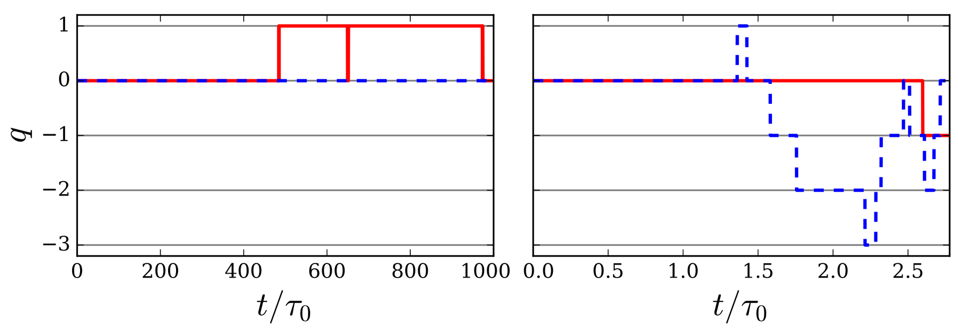
Motivation

Renewable energy sources are **scattered** and **fluctuating**. Their increasing penetration places the issue of **electrical grid stability** in the wider problem of stability of noisy coupled dynamical systems,

$$\{\text{electrical grid stability}\} \subset \{\text{perturbed dynamical systems}\}\,.$$

We assess the time needed for a dynamical system to be destabilized, based on the noise's parameters. In our approach, a larger amount of **inertia** in the system **does not stabilize** it.



Escape time

According to DeVille [2],

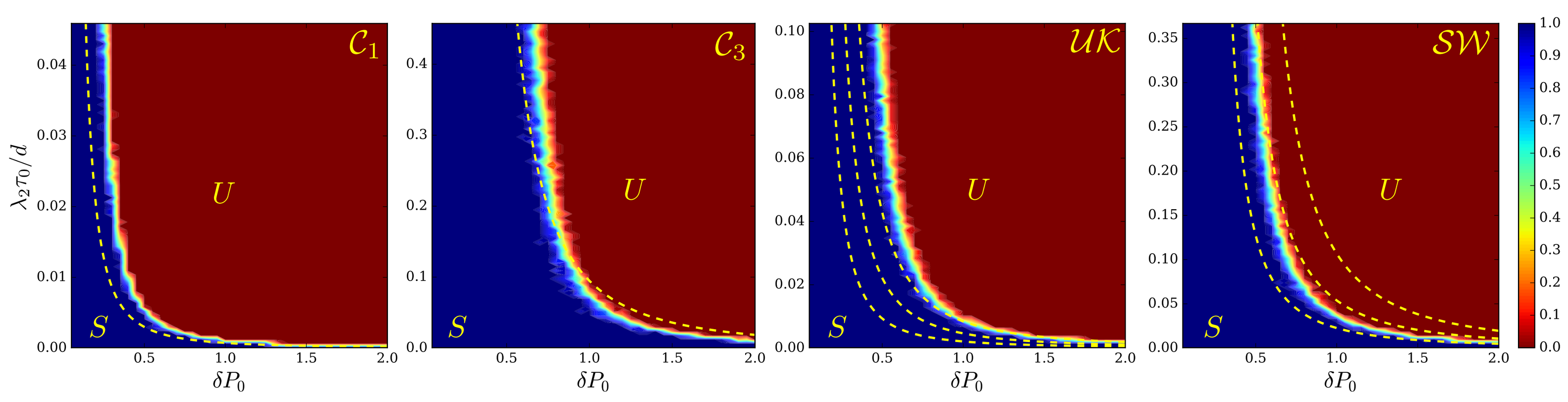
> **escapes from a basin of attraction occur in a neighborhood of a 1-saddle φ of the dynamics Eq.** (†).

Defining λ_α and $\mathbf{u}_\alpha$ the **eigenvalues** and **eigenvectors** of $\mathbb{L}$, one can solve Eq. (‡) [1], and compare the long time behavior of the angle displacements with the distance $\Delta := \|\boldsymbol{\theta}^{(0)} - \boldsymbol{\varphi}\|_2$,

$$\lim_{t\to\infty}\langle\delta\boldsymbol{\theta}^2(t)\rangle = \delta P_0^2 \sum_{\alpha\geq 2}\frac{\tau_0 + m/d}{\lambda_\alpha(\lambda_\alpha\tau_0 + d + m/\tau_0)} \leq \Delta^2\,, \qquad (\S)$$

giving an estimate of the **parameter domain** where the system is unlikely to be destabilized. The long time **typical excursion size** depends on the three **time scales**

noise: τ_0, **oscillators:** $\frac{m}{d}$, **network:** $\frac{d}{\lambda_\alpha}$.



Simulations of Eq. (†) with $m = 0$ were performed for a range of values for δP_0 and τ_0, recording the number of them that escaped the initial basin of attraction after a given number of iterations $T_{\rm obs}$. The noise sequences $\delta P_i(t)$ were generated.

The parameter space is then splitted in a region U where **all simulations escape** and a region S **where all simulations remain** in the basin.

> **The criterion Eq.** (§) **gives a good parametric estimate of the boundary between the regions** U **and** S.

Remarkably, the following asymtotics does not depend on inertia,

$$\lim_{t\to\infty}\langle\delta\boldsymbol{\theta}^2(t)\rangle = \begin{cases}\dfrac{\delta P_0^2\tau_0}{nd}Kf_1\,, & \tau_0 \ll \frac{d}{\lambda_\alpha}\,,\ \frac{m}{d}\,,\\[2ex] \dfrac{\delta P_0^2}{n}Kf_2\,, & \tau_0 \gg \frac{d}{\lambda_\alpha}\,,\ \frac{m}{d}\,.\end{cases}$$

The networks considered are:

- $\mathcal{C}_1$ – The cycle of length $n = 83$ vertices;
- $\mathcal{C}_3$ – The cycle with first- and third-neighbors with $n = 83$ vertices;
- $\mathcal{UK}$ – The UK transmission network composed of $n = 120$ vertices and 165 edges;
- $\mathcal{SW}$ – A small world network with $n = 200$ vertices.

Remark. *The value of Δ is obtained analytically for the cycle $\mathcal{C}_1$ and estimated numerically for $\mathcal{C}_3$, $\mathcal{UK}$, and $\mathcal{SW}$, see [3] for more details.*

References

[1] M. Tyloo, T. Coletta, and P. Jacquod, *Phys. Rev. Lett.* **120** (2018).
[2] L. DeVille, *Nonlinearity* **25** (2012).
[3] M. Tyloo, R. Delabays, and P. Jacquod, *under preparation* (2019).

The model

We consider the second-order system of n coupled oscillators, which represent the **swing equations** in the lossless line approximation,

$$m\ddot{\theta}_i + d\dot{\theta}_i = P_i(t) - \sum_{j=1}^{n} b_{ij}\sin(\theta_i - \theta_j)\,, \qquad (\dagger)$$

- $\theta_i \in (-\pi, \pi]$, $i = 1, ..., n$, are the oscillators angles;
- m, d are respectively the inertia and damping of each oscillator;
- $P_i(t)$, $i = 1, ..., n$, are the time-varying natural frequencies, or power injections/consumptions;
- b_{ij} are the elements of the weighted adjacency matrix of the interconnection graph.

Decomposing $\boldsymbol{P} = \boldsymbol{P}^{(0)} + \delta\boldsymbol{P}(t)$ and $\boldsymbol{\theta} = \boldsymbol{\theta}^{(0)} + \delta\boldsymbol{\theta}(t)$ and linearizing Eq. (†), one gets

$$m\delta\ddot{\boldsymbol{\theta}} + d\delta\dot{\boldsymbol{\theta}} = \delta\boldsymbol{P} - \mathbb{L}\left(\{\theta_i^{(0)}\}\right)\delta\boldsymbol{\theta}\,, \quad \text{where} \quad \mathbb{L}_{ij} = \begin{cases}-b_{ij}\cos(\theta_i - \theta_j)\,, & i \neq j\,,\\ \sum_{k\neq i}\cos(\theta_i - \theta_k)\,, & i = j\,,\end{cases} \qquad (\ddagger)$$

is a **weighted Laplacian matrix**.

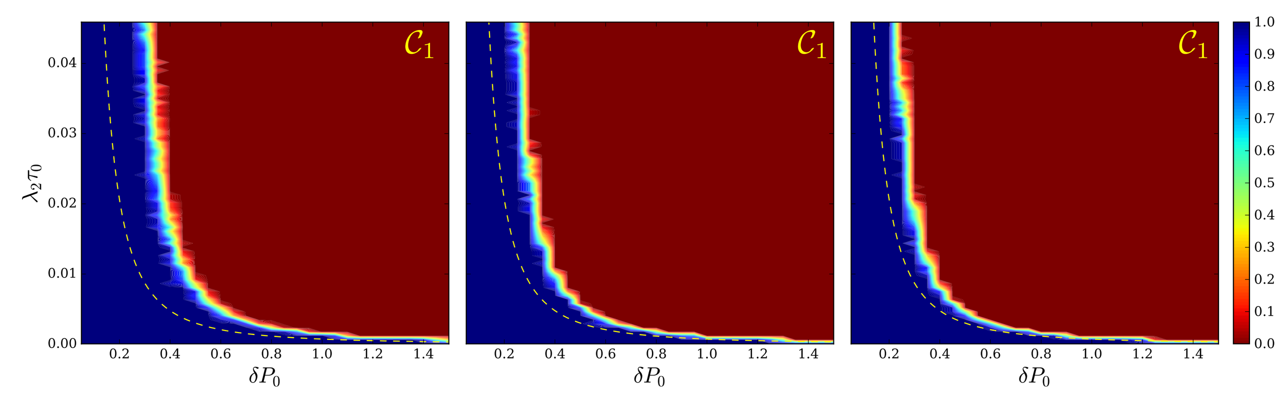
We apply an **additive random colored noise** to all natural frequencies,

$$\langle\delta P_i(t)\cdot\delta P_j(t')\rangle = \delta_{ij}\cdot\delta P_0^2\cdot e^{-|t-t'|/\tau_0}\,,$$

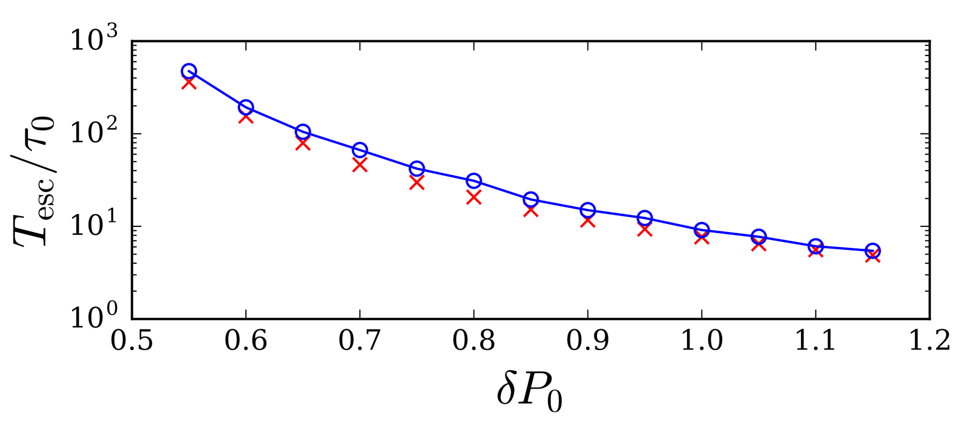
with δP_0 the noise's amplitude and τ_0 the decorrelation time. The noise is **time-correlated** and **independent in space**.

Superexponential escape time

Increasing the observation time $T_{\rm obs}$, we see the number of escape increasing.



Fixing $\tau_0 = 1.5$, we observe (blue circles) that the escape time increases **superexponentially** as δP_0 is decreased.



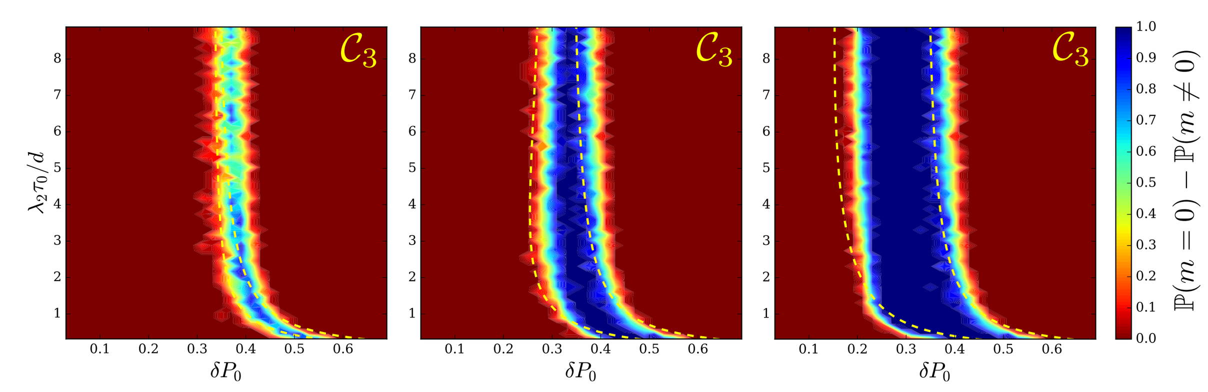
This fact is explained by observing that after a long enough time, the angle deviations $\delta\theta_i$ follow a **normal distribution** $\mathcal{N}(0, \bar{\sigma})$. Large excursions leading to basin escapes are then rare events, appearing in the distribution tails. The time needed to see such large excursion is estimated as

$$T_{\rm esc} \approx \left[2\int_{\beta\Delta}^{\infty}\mathbb{P}(\delta\bar{\theta})\mathrm{d}(\delta\bar{\theta})\right]^{-1}\,,$$

which is superexponential (red crosses).

Inertia

Comparing the cases $m > 0$ and $m = 0$, our analytical prediction and the simulations both conclude that **inertia almost always destabilizes the system**.



In the context of electrical network, however, the value of τ_0 is very large compared to the time scales of the network. Such system then evolves in a parameter region where the difference is negligible.

Conclusion

We proposed a method to assess the time needed for a system to leave its basin of attraction. This criterion is efficient to compute as it mainly relies on the inversion of a Laplacian matrix.

Under our assumptions, for a sufficiently long time, any system ends up escaping its basin. But the time needed for this increases superexponentially, exceeding any realistic time for any practical application.

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