Multistability in electric power grids on meshed, complex networks

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Joint work with

M. Tyloo, T. Coletta, and Ph. Jacquod

R. Delabays, M. Tyloo, and Ph. Jacquod, *Chaos* **27** (2017)
T. Coletta, R. D., I. Adagideli, and Ph. Jacquod, *New J. Phys* **18** (2016)

Hes·so VALAIS WALLIS
School of Engineering

Noisy injection

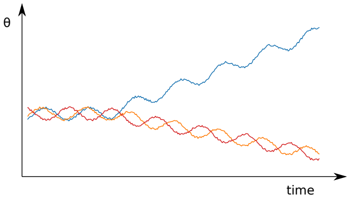
Energy Transition

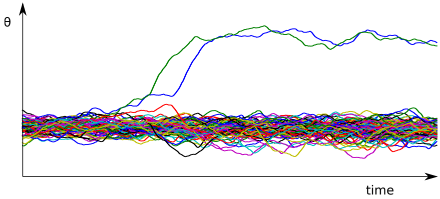
↓

Fluctuating power injections

↓

Possible loss of synchrony or convergence to a new synchronous state





K. Schmietendorf, J. Peinke, and O. Kamps, *Euro. Phys. J. B* **90** (2017)

B. Schäfer et al., *Phys. Rev. E* **95** (2017)

Questions

How likely is it for a system to lose synchrony?

Can we quantify this probability?

What are the relevant parameter of the fluctuations?

The model

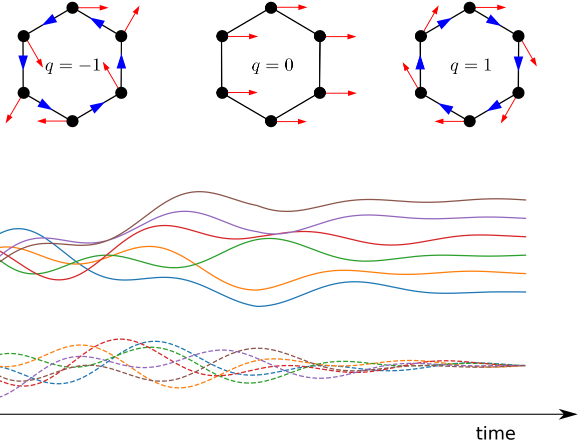
First-order, lossless approximation of the Swing Equations.

The Kuramoto model:

$$\dot{\theta}_i = (P_i + \delta P_i(t)) - K \sum_{j=1}^{n} a_{ij} \sin(\theta_i - \theta_j)\,, \qquad i \in \{1, \ldots, n\}\,.$$

- $\theta_i \in \mathbb{S}^1$: Voltage angle,
- $P_i \in \mathbb{R}$: Power injection,
- K: Identical coupling,
- a_{ij}: Adjacency matrix.

Multistability

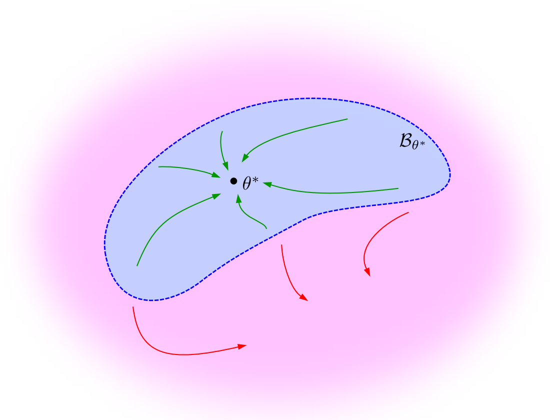


R. D., T. Coletta, and Ph. Jacquod, *J. Math. Phys.* **57** (2016)

R. D., T. Coletta, and Ph. Jacquod, *J. Math. Phys.* **58** (2017)

Basins of attraction

$$\mathcal{B}_{\theta^*} := \{\theta^\circ \in \mathbb{T}^n \ : \ \theta(0) = \theta^\circ \implies \theta(t \to \infty) = \theta^*\} \ .$$



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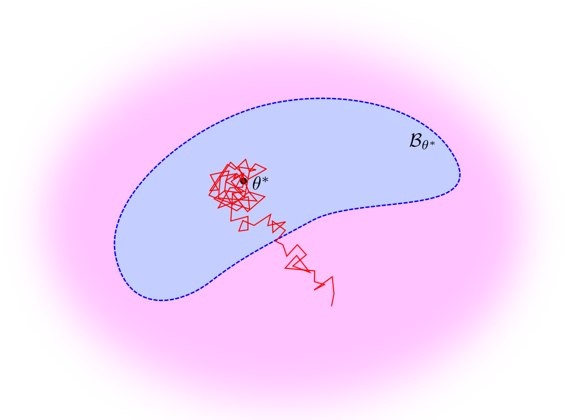

Simulations

Gaussian time-correlated noise: $\delta P_i \colon [0, +\infty) \to \mathbb{R}$ such that

$$\left\langle \delta P_i(t) \cdot \delta P_j(t') \right\rangle = \delta_{ij} \cdot \delta P_{i,0} \cdot \delta P_{j,0} \cdot \exp(-|t - t'|/\tau_0)\,,$$

where $\tau_0 > 0$ is the correlation time.

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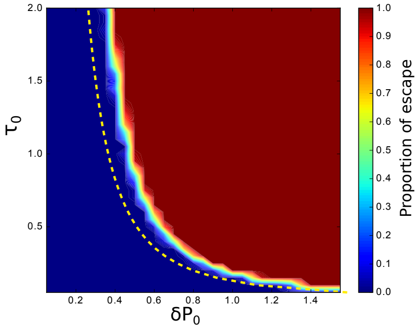

- Cycle $n = 83$,
- Identical frequencies: $P_i \equiv 0$,
- Observation time: 8e5 iterations,
- Dashed yellow line: our prediction.

Analytical prediction

We consider

$$\theta_i(t) = \theta_i^* + \delta\theta_i(t)\,, \qquad \mathcal{J}(\boldsymbol{\theta}^*)\cdot \mathbf{u}_\alpha = \lambda_\alpha \mathbf{u}_\alpha\,.$$

M. Tyloo, R. D., and Ph. Jacquod, *under preparation*

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Linearizing the dynamics around the stable fixed point:

$$\langle \delta\boldsymbol{\theta}^2\rangle = \left\langle \left(\sum_\alpha c_\alpha \mathbf{u}_\alpha\right)^2 \right\rangle$$

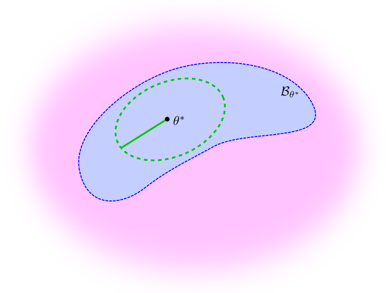
$$= \sum_{\alpha\geq 2,i} (\delta P_{i,0}\, u_{\alpha,i})^2 \left(\frac{1}{\lambda_\alpha(\lambda_\alpha+\tau_0^{-1})} + \frac{e^{-2\lambda_\alpha t}}{\lambda_\alpha(\lambda_\alpha-\tau_0^{-1})} - \frac{2e^{-(\lambda_\alpha+\tau_0^{-1})t}}{\lambda_\alpha^2-\tau_0^{-2}} \right)$$

M. Tyloo, R. D., and Ph. Jacquod, *under preparation*

Analytical prediction

For $\delta P_{i,0} \equiv \delta P_0$ and $t \to \infty$,

$$\left\langle \delta\boldsymbol{\theta}^2 \right\rangle = \delta P_0^2 \sum_{\alpha \geq 2} \frac{1}{\lambda_\alpha(\lambda_\alpha + \tau_0^{-1})} \, .$$



M. Tyloo, R. D., and Ph. Jacquod, *under preparation*.

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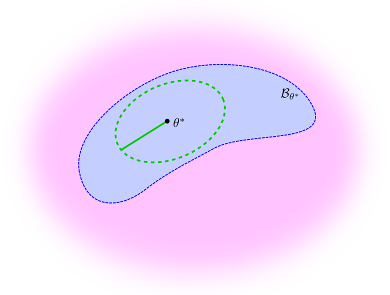

Can we derive a condition for loss of synchrony?

M. Tyloo, R. D., and Ph. Jacquod, *under preparation*.

Escape from the basin

Gradient dynamics:

$$\dot{\boldsymbol{\theta}} = -\nabla V(\boldsymbol{\theta}), \quad V(\boldsymbol{\theta}) = -\sum_i P_i \theta_i + \sum_{i<j} a_{ij} \left(1 - \cos(\theta_i - \theta_j)\right) .$$

L. DeVille, *Nonlinearity* **25** (2012)

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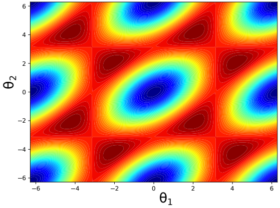
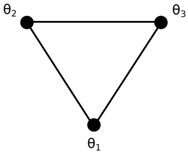
Almost surely, escape occurs through a 1-saddle.





L. DeVille, *Nonlinearity* **25** (2012)

Single cycle

Distance between stable sync state $\boldsymbol{\theta}^{(0)}$ and the closest 1-saddle $\boldsymbol{\varphi}^{(1)}$ is

$$\|\boldsymbol{\theta}^{(0)} - \boldsymbol{\varphi}^{(1)}\|_2^2 = \frac{n(n^2-1)}{12(n-2)^2}\pi^2 =: \Delta\,.$$

R. D., M. Tyloo, Ph. Jacquod, *Chaos* **27** (2017)

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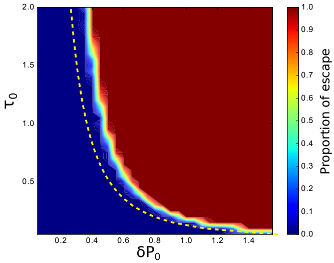
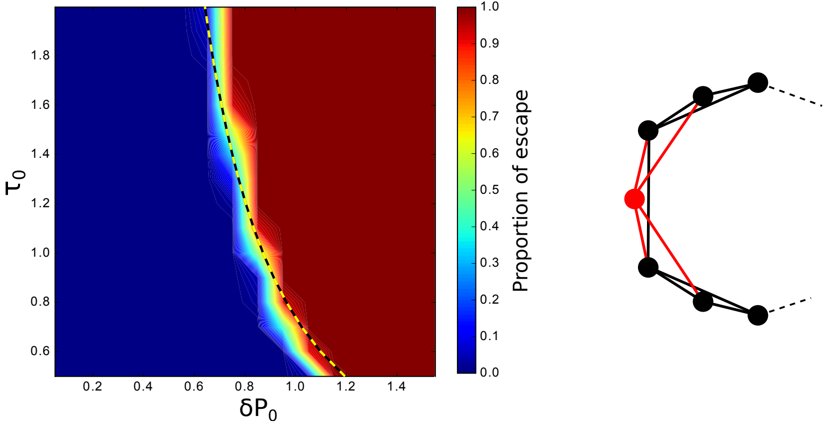
$$\delta P_0^2 \sum_{\alpha\geq 2} \frac{1}{\lambda_\alpha(\lambda_\alpha + \tau_0^{-1})} = \Delta\,.$$

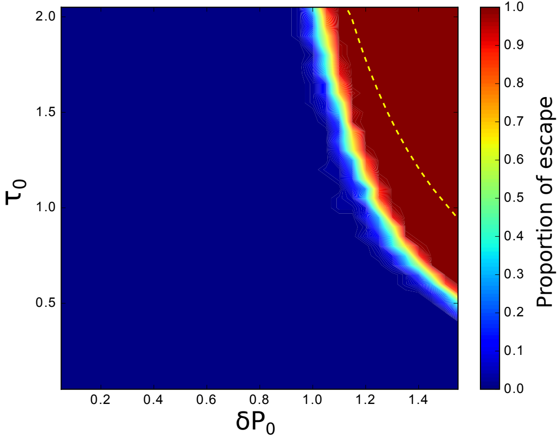


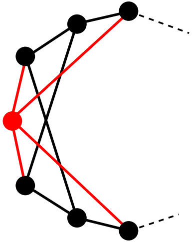
R. D., M. Tyloo, Ph. Jacquod, *Chaos* **27** (2017)

Cycle with 2^{nd} neighbor, $n = 83$

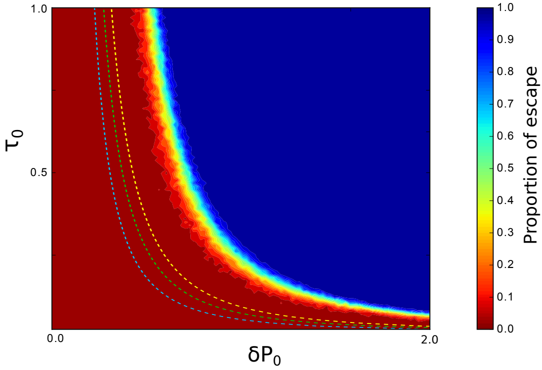


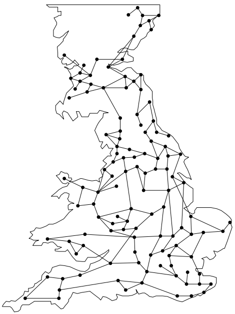
Cycle with 3rd neighbor, $n = 83$



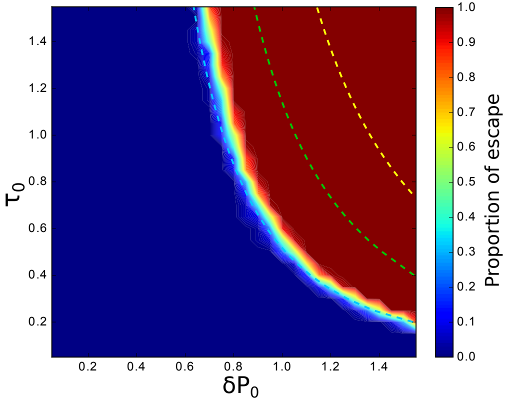


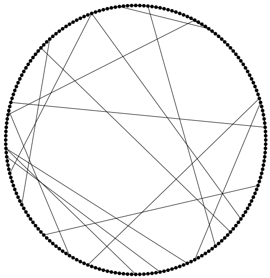
Pseudo-UK, $n = 120$



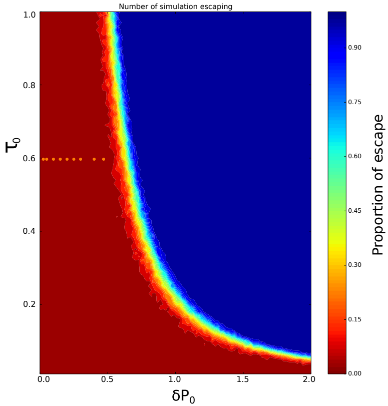


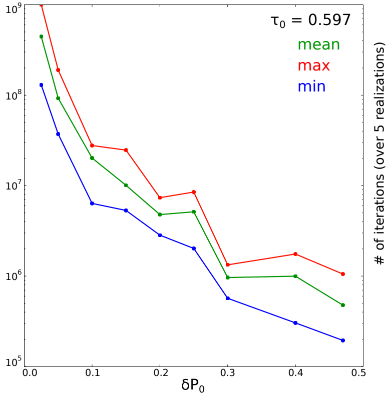
Small-world, $n = 200$





Caveat – UK network





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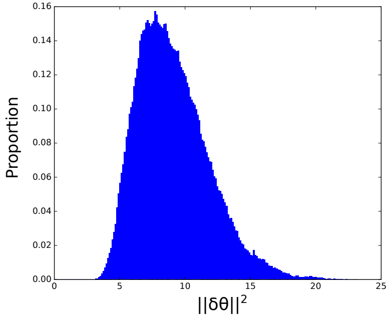

Conclusion

- Qualitatively the same for the second order model

$$\ddot{\theta}_i + \gamma\dot{\theta}_i = (P_i + \delta P_i(t)) - K\sum_j a_{ij}\sin(\theta_i - \theta_j)\,.$$

- Where are PV and Wind on this diagram?

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Thank you!

Histograms of 2-norms