Topologically Protected Loop Flows in High Voltage AC Power Grids

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1: Power flow equations

Steady state, active power flows in AC electrical networks are governed by

$$P_l = \sum_m \Big(B_{lm}|V_l||V_m|\sin(\theta_l - \theta_m) + G_{lm}|V_l|[|V_l| - |V_m|\cos(\theta_l - \theta_m)] \Big)\,, \qquad (1)$$

- $V_l = |V_l|e^{i\theta_l}$ complex voltage at node l.
- $P_l \geq 0\,(\leq 0)$ injected (consumed) active power at node l.
- B_{lm} and G_{lm} susceptance and conductance of the line connecting nodes l and m.

Working assumptions

- Consider PV-nodes and neglect voltage fluctuations $|V_l| = 1$.
- High voltage AC transmission lines have $G_{lm}/B_{lm} \approx 5\% - 10\%$.
  Neglecting the conductance (lossless approximation), the active power flow on a line is $P_{lm} = B_{lm}\sin(\theta_l - \theta_m)$.
  In this limit, analogy with the DC Josephson current between superconducting islands.

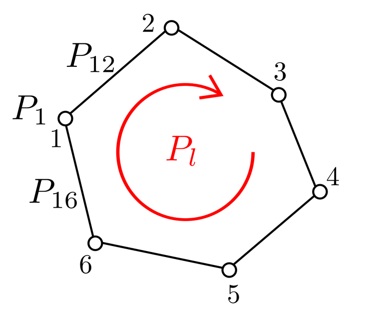
Motivation:

- **1GW of power circulating around lake Erie (USA) without being delivered anywhere [1, 2].**
- **For a fixed set of power injections and meshed networks, multiple solutions to the power flow problem exist [3]. They are related to each other by power circulating in closed loops of the network: Loop Flows.**
- **How are circulating power flows created? and why are they so robust?**

2: Multi-stability & Vortex flows

In the lossless approximation, $G_{lm} = 0$, the presence of cycles in a network is the ingredient necessary for the existence of multistable solutions of the power flows Eq. (1).

- Any two solutions of the lossless power flow problem are related by a collection of loop flows [4, 5].
- The kernel of the incidence matrix of the network spans the space of different solutions (Kirchoff conservation of power flows).
- The complex voltage around any loop is single valued. The allowed values of circulating power are discrete and indexed by integer winding numbers



$P_1 = P_{12} + P_{16}$
$P_1 = P_{12} + P_l + P_{16} - P_l$

$$q_\alpha = (2\pi)^{-1}\sum_{l=1}^{n_\alpha} |\theta_{l+1} - \theta_l| \in \mathbb{Z}\,, \qquad (2)$$

- q_α counts the number of times the complex voltage winds around the origin in the complex plane as one goes around the loop α.

Open issues investigated:

- **Existence and resilience of loop flows for $0 < G_{lm} \ll B_{lm}$.**
- **Dissipation in vortex flow carrying solutions.**
- **Mechanisms for vortex flow creation.** Untwisting the complex voltage V_l cannot be done smoothly without driving $|V_l| \to 0$ somewhere. Topological protection!

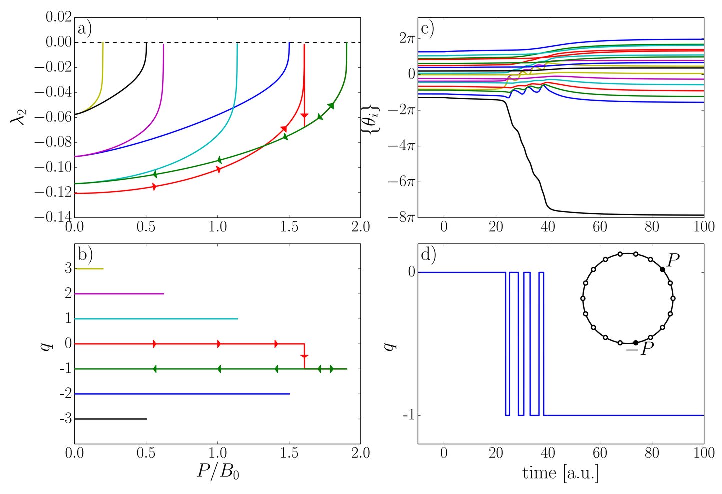
3: Mechanisms for vortex flow creation

Loss of stability of vortex-free states

- The swing equations describe the dynamics of voltage angles θ_l

$$\dot{\theta}_l = P_l - \sum_m B_{lm}\sin(\theta_l - \theta_m) + G_{lm}[1 - \cos(\theta_l - \theta_m)]\,. \qquad (3)$$

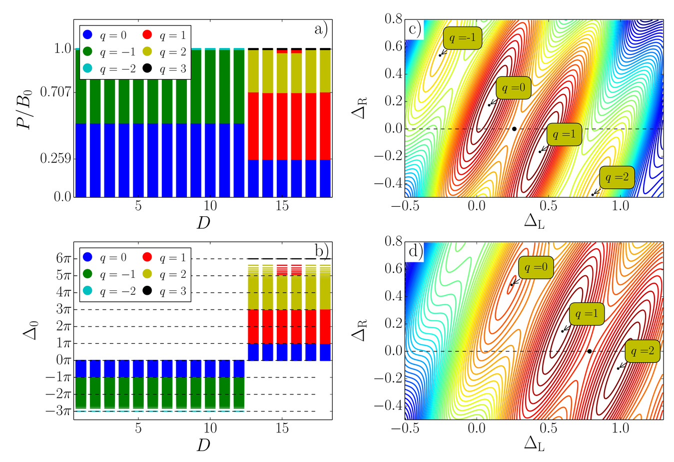
- The linear stability of solutions of the power flow equations is determined by the Lyapunov spectrum $\{\lambda_i\}$ of Eq. (3).



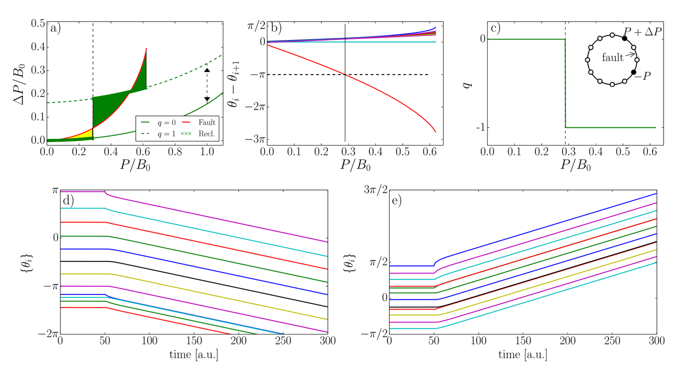
- Upon increasing power injections, vortex-free states can become unstable before vortex-carrying solutions in asymmetric cycles.
- The loss of stability ($\lambda_2 \to 0$) is accompanied by a transient during which the winding number changes value. (Check out movie!)
- Decreasing the power injection back to the original values does not bring back the system into a vortex-free state. Hysteretic behavior resulting from the topological protection brought about by the integer winding number.
- The loss of stability generally occurs for phase differences exceeding $\pi/2$. Thus, this mechanism is not of direct relevance for electric transmission grids where thermal limits of lines impose $|\theta_l - \theta_m| \lesssim \pi/6$.

Line tripping and reclosing [6]

- The transient following line tripping can drive the system's operating state into the basin of attraction of a vortex-carrying state.



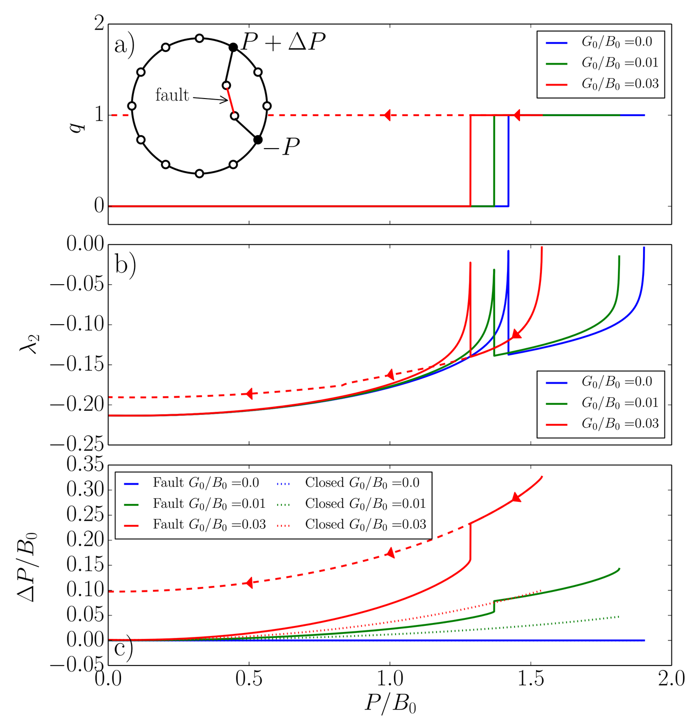
- For single loop networks having only one consumer and one producer separated by non injecting nodes, the basins of attraction of solutions having different winding numbers can be visualized by plotting a projected version of the Lyapunov function.
- Saddle points of the Lyapunov function occur when the phase difference on the reclosed line is equal to $(2k+1)\pi$, $k \in \mathbb{Z}$.
- This picture holds for small but finite values of conductance.



- Relevant for electrical networks: ensuring "N-1 feasibility" does not prevent the creation of loop-flow carrying states.

Tripping a line traversing a loop

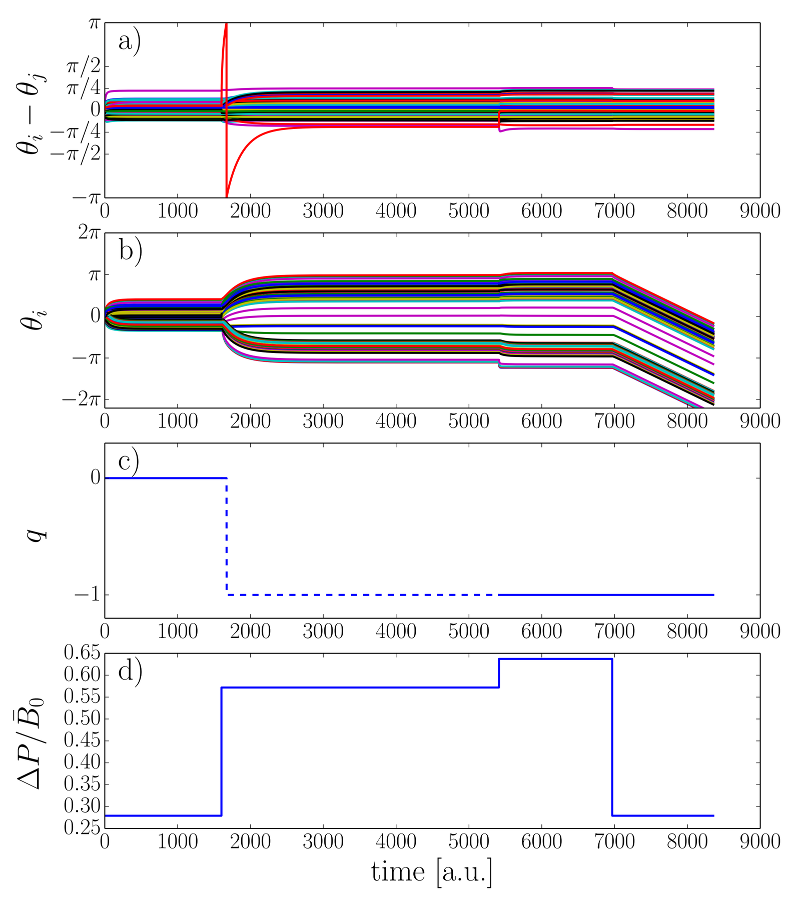
- In an asymmetrical double loop network, initially prepared in a state having no vortex flow on any of its loops, power is distributed along the three different paths.
- Depending on the operating conditions (amount of power injected/consumed), the transient following the tripping of the line traversing the central loop can generate a vortex flow on the resulting large cycle.

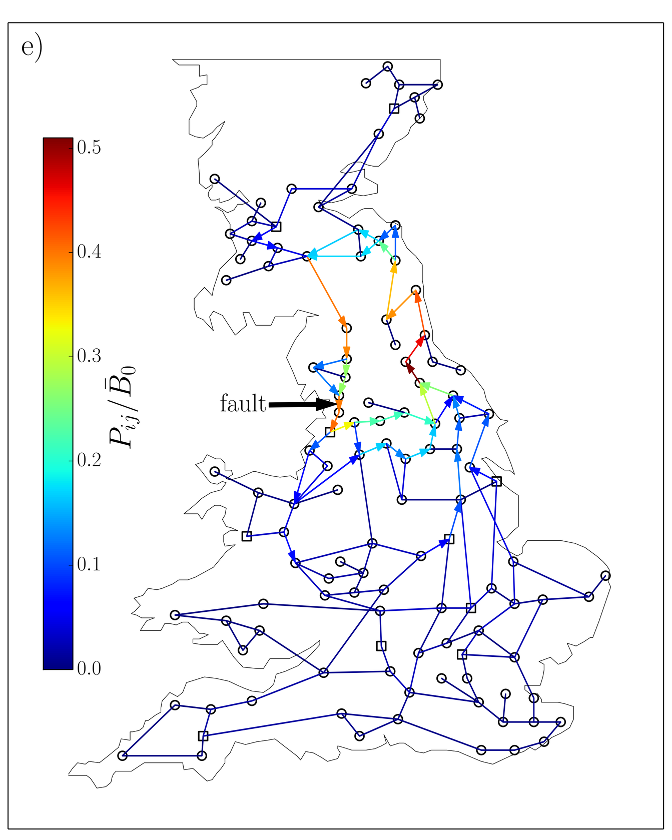


- Active power transmission losses are significantly higher in the vortex-carrying state.
- The parameter range over which the vortex-carrying state is stable decreases for larger conductance to susceptance ratios.

4: Vortex flows in realistic electric network topologies

- Network topology of the UK high-voltage AC power grid (10 generators, 110 consumers).
- Ratios $G_{lm}/B_{lm} = 0.1$.
- Large central loop prone to vortex-carrying states.
- Starting from a stable vortex-free solution, the system's operating state is perturbed by a line tripping. The system's state is then left to stabilize ("N-1 feasibility" satisfied) before reclosing the line.





- A vortex-carrying state is created during the line tripping and reclosing event.
- Total transmission losses in the vortex-carrying state are twice as large as in the initial vortex-free state.
- Reducing the power injections back to their original value prior of the fault, does not allow to get rid of the vortex flow but only reduces the synchronous frequency of the system.

5: Conclusion

We investigated how circulating power flows can be created and how they behave in the presence of ohmic dissipation. We showed how changing operating conditions may generate them, how significantly more power is ohmically dissipated in their presence and how they are topologically protected, even in the presence of dissipation, so that they persist when operating conditions are returned to their original values.

→ "N-1 feasibility" should be complemented with vortex-flow detection.
→ Line tripping and reclosing does not necessarily bring the system back to its original state.
→ Mesures to get rid of vortex flows? open cycles!
→ Open problem of number of possible power flow solutions in meshed networks (visit the poster by R. Delabays).

References

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