# Coherence and Concentration in Tightly-Connected Networks

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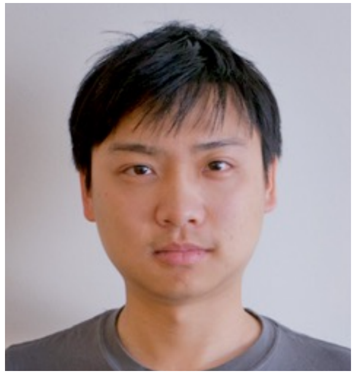
Johns Hopkins University

Data-based Diagnosis of Networked Dynamical Systems

CCS 2021 Satellite Symposium

November 3, 2021

# Acknowledgements



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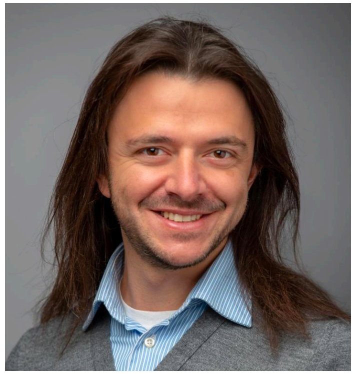

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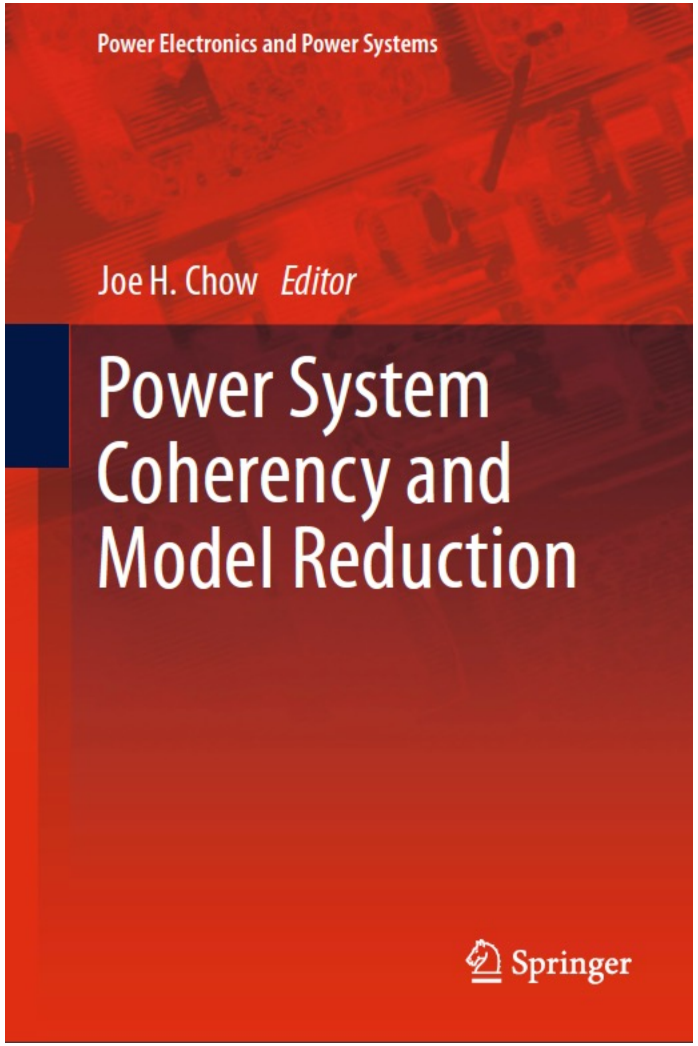
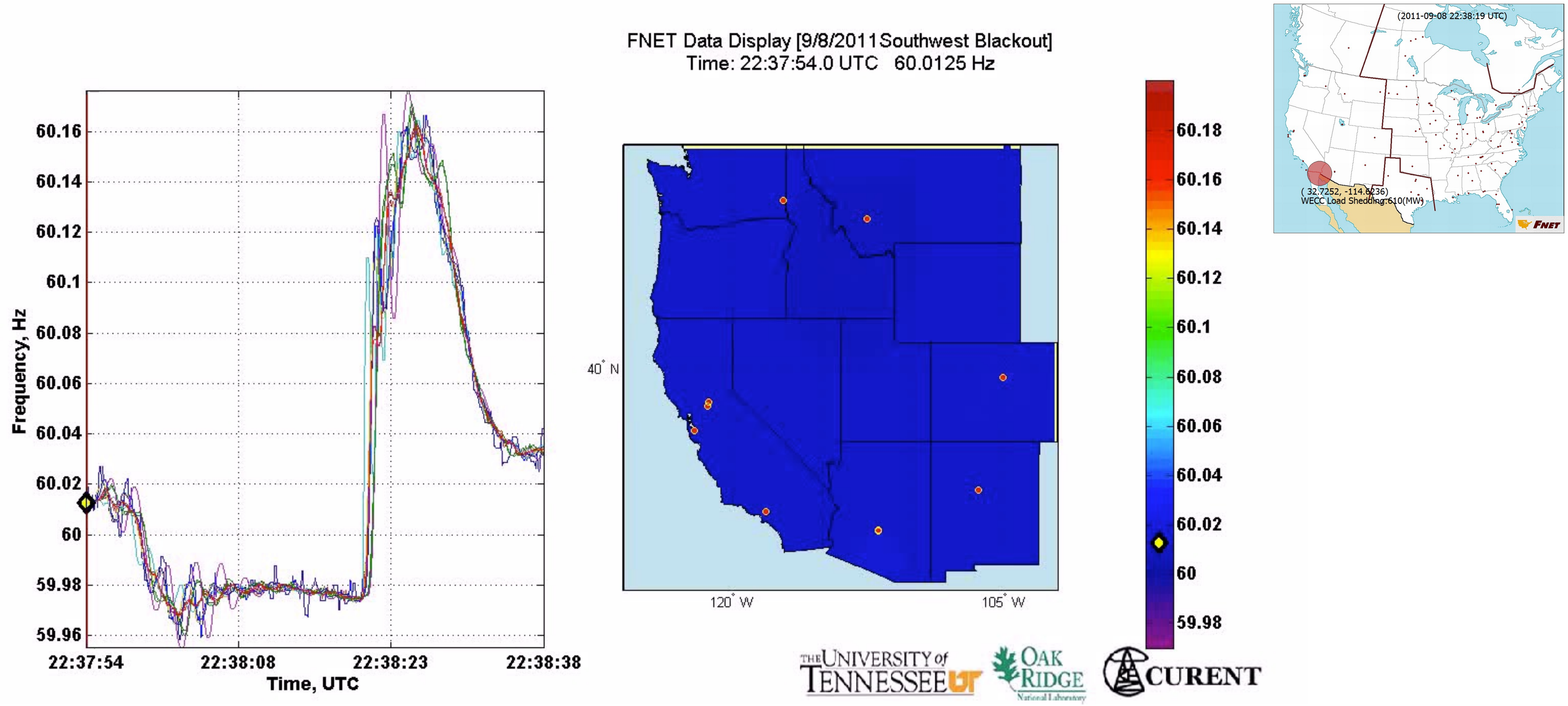
# Coherence in Power Networks

- Studied since the 70s
  - Podmore, Price, Chow, Kokotovic, Verghese, Pai, Schweppe,…
- Enables aggregation/model reduction
  - Speed up transient stability analysis
- Many important questions
  - How to identify coherent modes?
  - How to accurately reduce them?
  - What is the cause?
- Many approaches
  - Timescale separations (Chow, Kokotovic,)
  - Krylov subspaces (Chaniotis, Pai '01)
  - Balanced truncation (Liu et al '09)
  - Selective Modal Analysis (Perez-Arriaga, Verghese, Schweppe '82)



# This talk




**Goals:**

1. **Characterize the coherence response from a frequency domain perspective**
2. **Leverage the coherence response to obtain accurate reduced order models**

Coherence and Concentration in Tightly-Connected Networks

Hancheng Min and Enrique Mallada

*ArXiv preprint: arXiv:2101.00981*

Accurate Reduced-Order Models for Heterogeneous Coherent Generators

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Storage-Based Frequency Shaping Control

Yan Jiang, Eliza Cohn, Petr Vorobev, *Member, IEEE*, and Enrique Mallada, *Senior Member, IEEE*

**[TPS 21]** ***IEEE Transactions on Power Systems**, 2021*

**Grid-forming frequency shaping control**

Yan Jiang[1], Andrey Bernstein[2], Petr Vorobev[3], and Enrique Mallada[1]

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# Outline

- Characterization of Coherent Dynamics [Min, M ‘21]
- Reduced-Order Model of Coherent Response [Min, Paganini, M ‘21]

Coherence and Concentration in Tightly-Connected Networks

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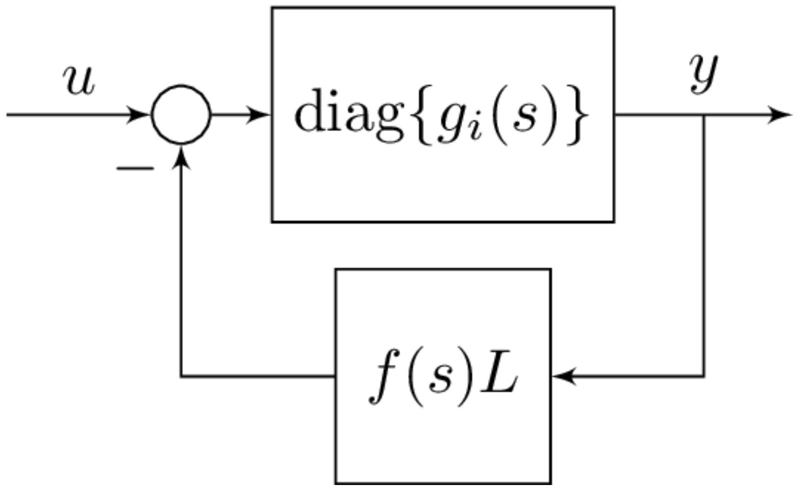
# Coherence in networked dynamical systems

**Block Diagram:**




Node dynamics: $g_i(s), i = 1, 2, \cdots, n$

Symmetric Real Network Laplacian: $L$

$$L = V\Lambda V^T, \; V = [\mathbb{1}/\sqrt{n}, V_\perp]$$
$$\Lambda = \mathrm{diag}\{0, \lambda_2(L), \cdots, \lambda_n(L)\}$$

Coupling dynamics: $f(s)$

**Examples:**

- Consensus Networks:

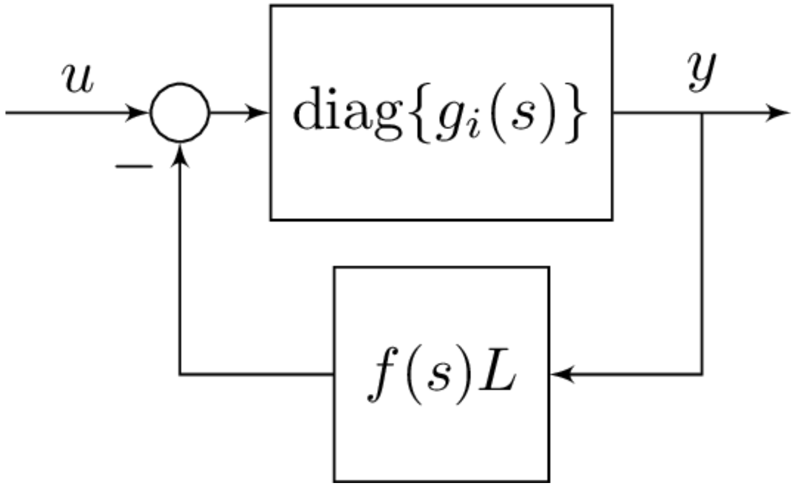
$$g_i(s) = \frac{1}{s}$$
$$f(s) = 1$$

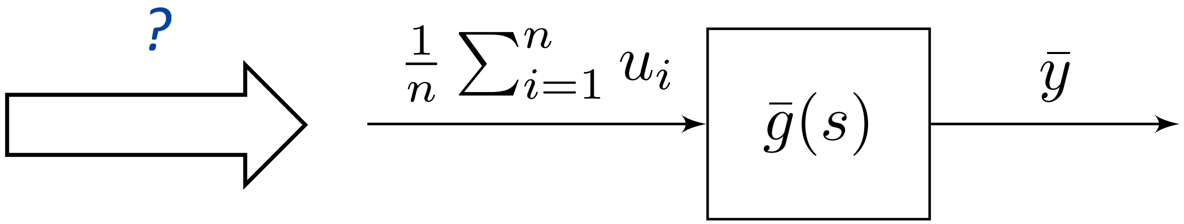
- Power Networks (2nd order generator):

$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}}$$
$$f(s) = \frac{1}{s}$$

# Coherence in networked dynamical systems

## Block Diagram:




*1.When does this network exhibit coherence?*

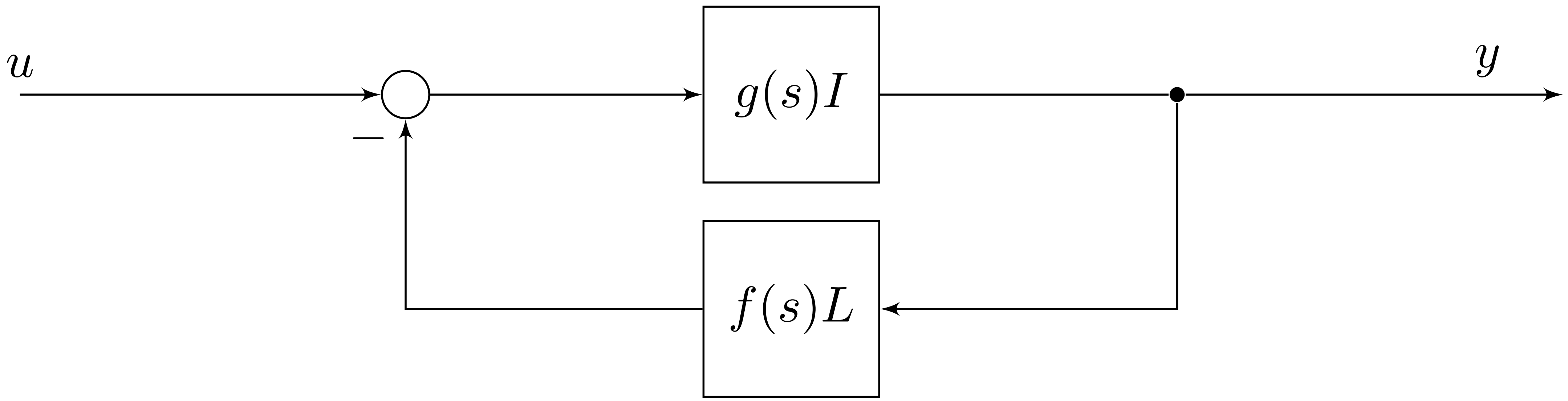
*2.What is the exact **coherent dynamics** of this network?*

1. Coherence can be understood as a **low rank** property the **closed-loop transfer matrix**
2. It emerges as the **effective algebraic connectivity** increases
3. The coherent dynamics is given by the **harmonic mean** of nodal dynamics

$$\bar{g}(s) = \left( \frac{1}{n} \sum_{i=1}^{n} g_i^{-1}(s) \right)^{-1}$$

# Network Coherence: Homogeneous Case

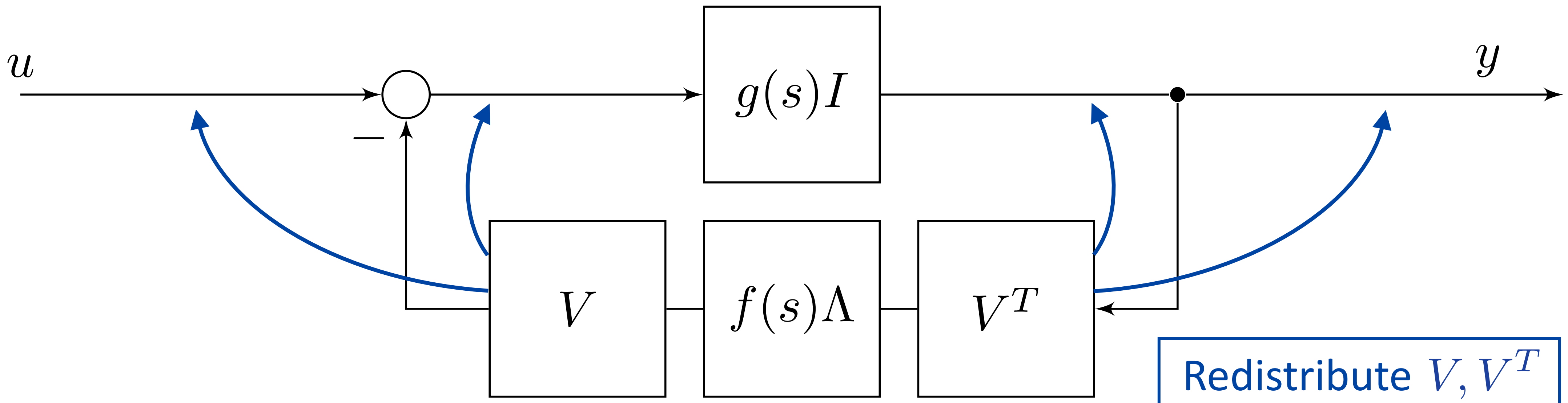
Assume homogeneity: $g_i(s) = g(s), \; i = 1, \cdots, n$




Eigendecomposition $L = V \Lambda V^T$

# Network Coherence: Homogeneous Case

Assume homogeneity: $g_i(s) = g(s),\ i = 1, \cdots, n$

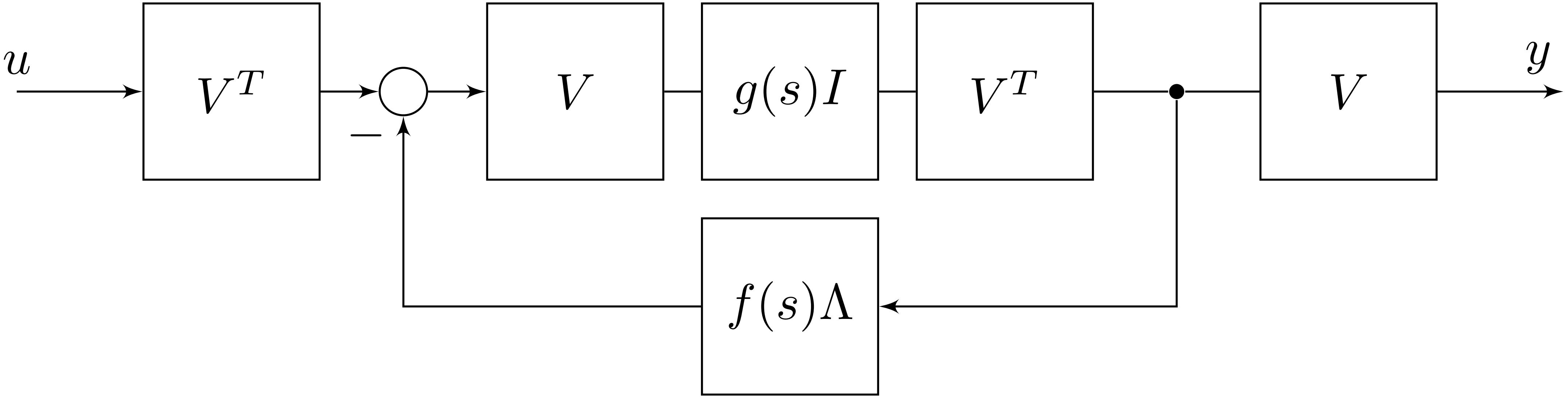



Redistribute $V, V^T$

# Network Coherence: Homogeneous Case

Assume homogeneity: $g_i(s) = g(s), \; i = 1, \cdots, n$

Merge forward path
$V^T V = I$



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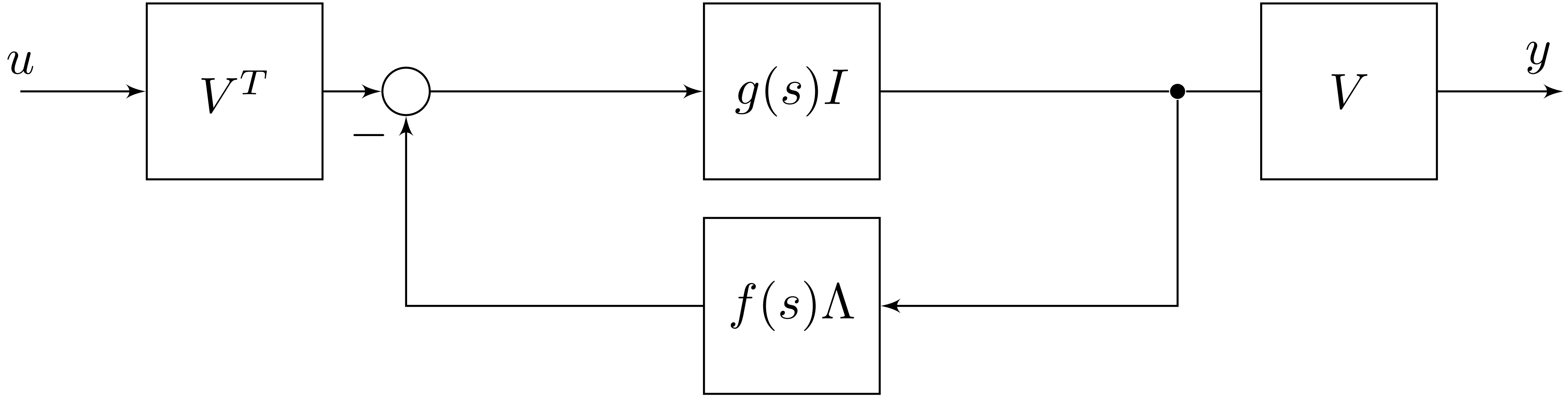

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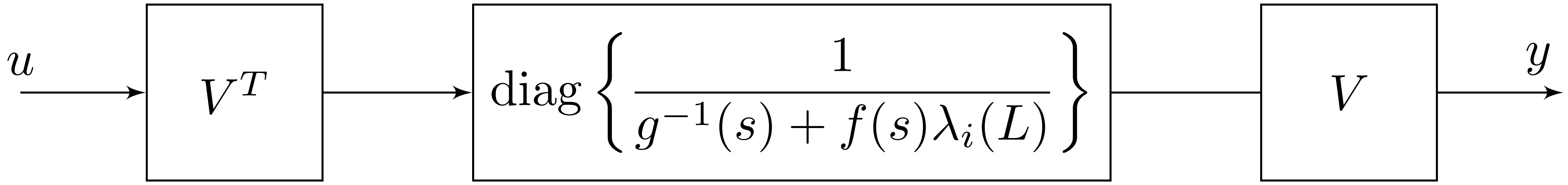


# Network Coherence: Homogeneous Case

Assume homogeneity: $g_i(s) = g(s),\ i = 1, \cdots, n$

The transfer matrix from input $u$ to output $y$ :

$$T(s) = V \operatorname{diag}\left\{\frac{1}{g^{-1}(s) + f(s)\lambda_i(L)}\right\}_{i=1}^{n} V^T$$

$$V = [\mathbb{1}/\sqrt{n}, V_\perp],\ \lambda_1(L) = 0$$

$$T(s) = \frac{1}{n} g(s) \mathbb{1}\mathbb{1}^T + V_\perp \operatorname{diag}\left\{\frac{1}{g^{-1}(s) + f(s)\lambda_i(L)}\right\}_{i=2}^{n} V_\perp^T$$

**Coherent dynamics** independent of the network structure

Dynamics dependent of the network structure

# Network Coherence: Homogeneous Case

$$T(s) = \frac{1}{n} g(s) \mathbb{1}\mathbb{1}^T + \boxed{V_\perp \operatorname{diag}\left\{ \frac{1}{g^{-1}(s) + f(s)\lambda_i(L)} \right\} V^T}$$

The effect of non-coherent dynamics vanishes as:

- The **algebraic connectivity** $\lambda_2(L)$ of the network increases

  For almost any $s_0 \in \mathbb{C}$

  $$\lim_{\lambda_2(L) \to +\infty} \left\| T(s_0) - \frac{1}{n} g(s_0) \mathbb{1}\mathbb{1}^T \right\| = 0$$

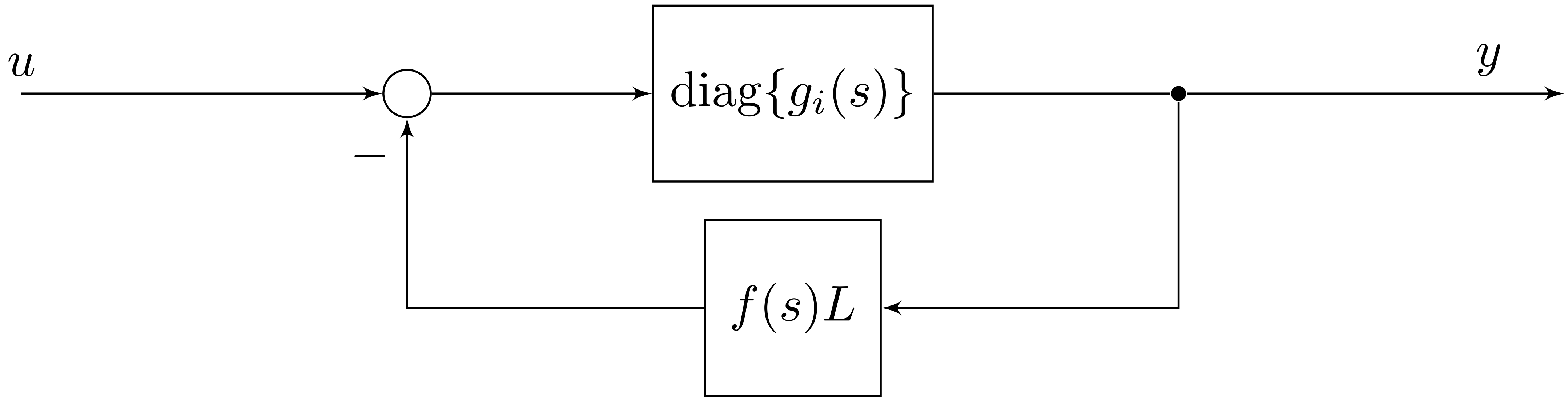
- The point of interest gets close to a **pole** of $f(s)$

  For $s_0 \in \mathbb{C}$, a pole of $f(s)$

  $$\lim_{s \to s_0} \left\| T(s) - \frac{1}{n} g(s) \mathbb{1}\mathbb{1}^T \right\| = 0$$

Our **frequency-dependent** coherence measure $\left\| T(s) - \frac{1}{n} g(s) \mathbb{1}\mathbb{1}^T \right\|$ is controlled by the **effective algebraic connectivity** $|f(s)|\lambda_2(L)$

# Network Coherence: Heterogeneous Case



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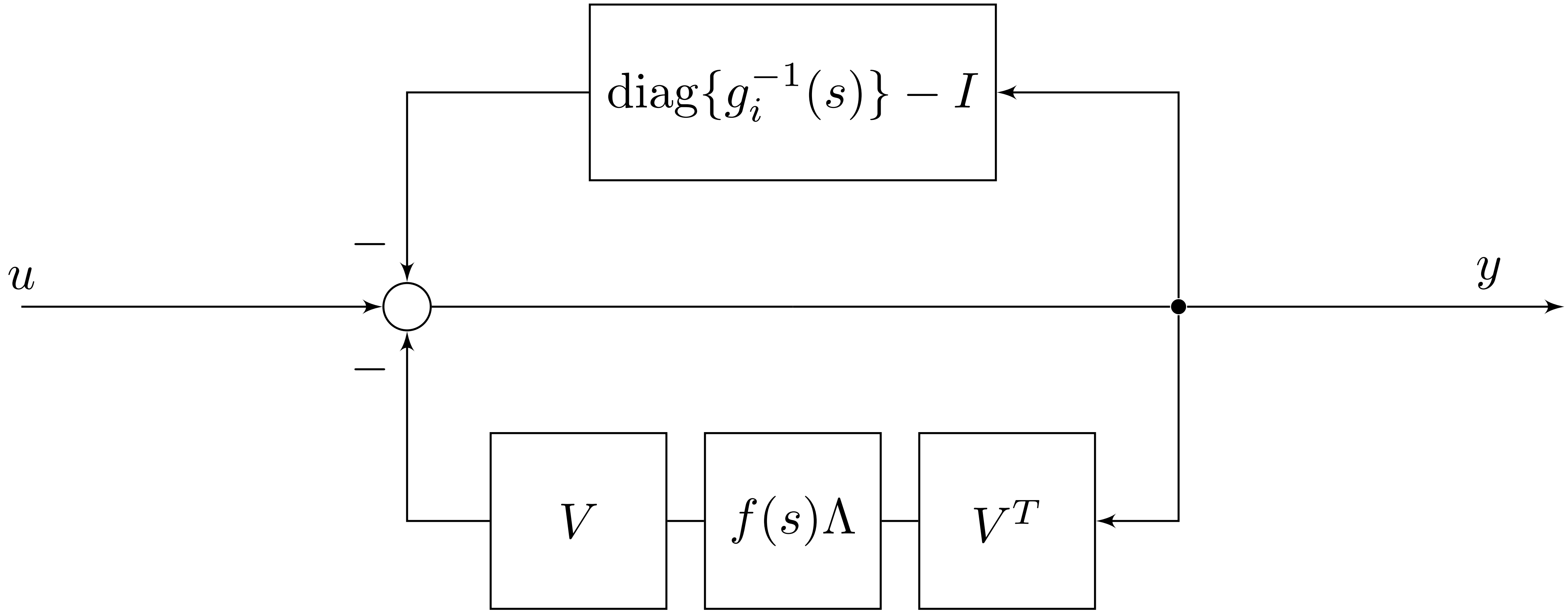

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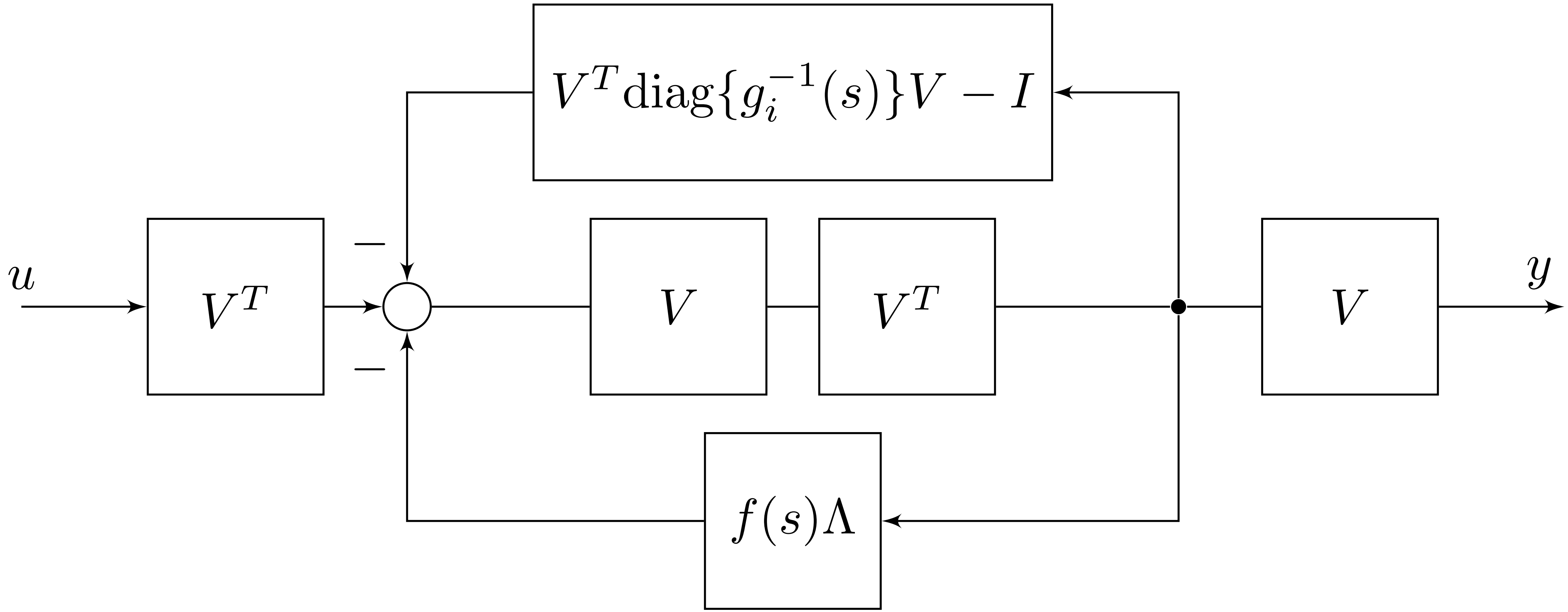

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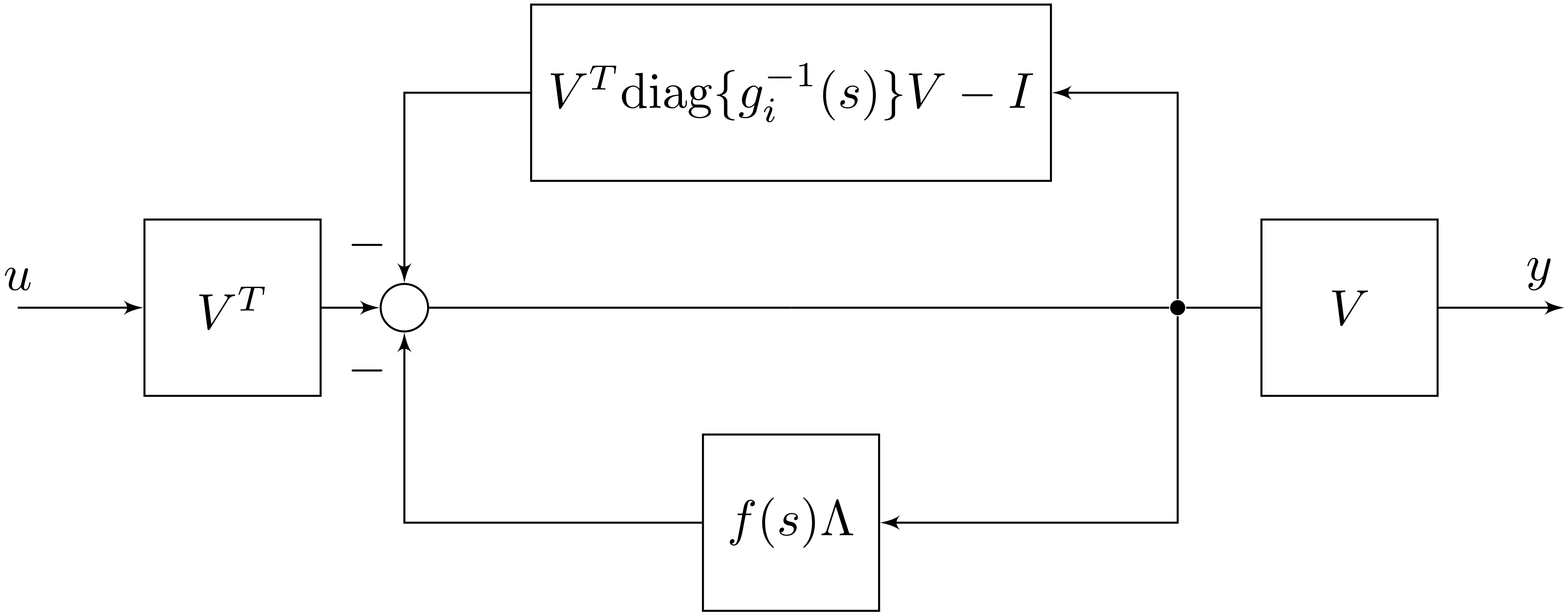
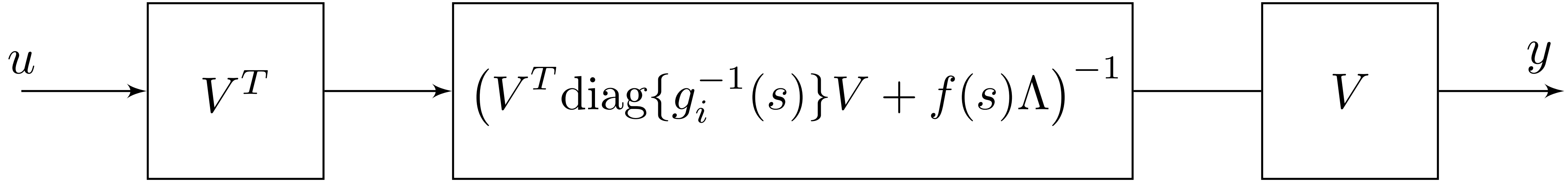


# Network Coherence: Heterogeneous Case




The transfer matrix from input $u$ to output $y$ :

$$T(s) = V\left(V^T \mathrm{diag}\{g_i^{-1}(s)\}V + f(s)\Lambda\right)^{-1} V^T$$

# Network Coherence: Heterogeneous Case

The transfer matrix from input $u$ to output $y$ :

$$T(s) = V\left(V^T \mathrm{diag}\{g_i^{-1}(s)\}V + f(s)\Lambda\right)^{-1} V^T$$

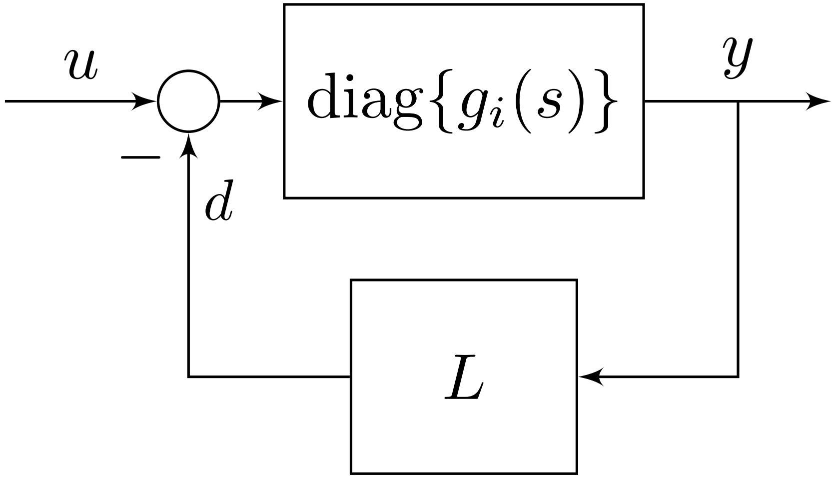
$$T(s) = \frac{1}{n}\bar{g}(s)\mathbb{1}\mathbb{1}^T + N(s)$$

**Coherent Dynamics?**

Network dependent?

# Informed guess for coherent dynamics: $\bar{g}(s)$

Block Diagram:




Dynamics for node $i$

$$y_i(s) = g_i(s)(u_i(s) - d_i(s)),\ i = 1, \cdots, n$$

Assume all nodes output are **identical** as the result of **coherence**

$$y_i(s) = \bar{y}(s)$$

$$g_i^{-1}(s)\bar{y}(s) = u_i(s) - d_i(s),\ i = 1, \cdots, n$$

Average equations from $i = 1$ to $n$:

$\mathbb{1}^T L = \mathbb{0}$

$$\left(\frac{1}{n}\sum_{i=1}^{n} g_i^{-1}(s)\right)\bar{y}(s) = \frac{1}{n}\sum_{i=1}^{n} u_i(s) - \boxed{\frac{1}{n}\sum_{i=1}^{n} d_i(s)}$$

=0

**Coherent Dynamics:**

$$\bar{y}(s) = \left(\frac{1}{n}\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1} \frac{1}{n}\sum_{i=1}^{n} u_i(s)$$

$$\bar{g}(s) = \left(\frac{1}{n}\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1}$$

Harmonic mean of all $g_i(s)$

# Network Coherence: Heterogeneous Case

$$T(s) = \frac{1}{n}\bar{g}(s)\mathbb{1}\mathbb{1}^T + \boxed{T(s) - \frac{1}{n}\bar{g}(s)\mathbb{1}\mathbb{1}^T} \qquad \bar{g}(s) = \left(\frac{1}{n}\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1}$$

The effect of non-coherent dynamics vanishes as:

- For almost any $s_0 \in \mathbb{C}$

$$\lim_{\lambda_2(L)\to+\infty} \left\| T(s_0) - \frac{1}{n}\bar{g}(s_0)\mathbb{1}\mathbb{1}^T \right\| = 0$$

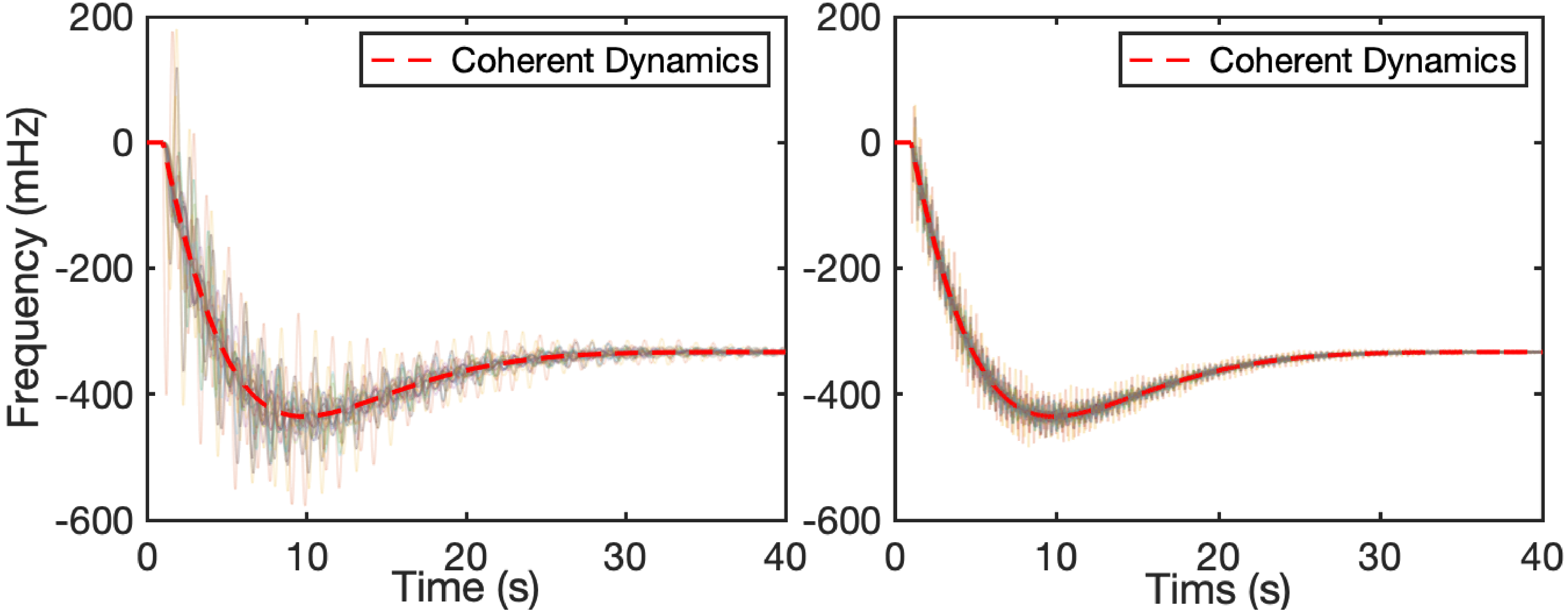
- For $s_0 \in \mathbb{C}$, a pole of $f(s)$

$$\lim_{s\to s_0} \left\| T(s) - \frac{1}{n}\bar{g}(s)\mathbb{1}\mathbb{1}^T \right\| = 0$$

- Excluding zeros: the limit holds at zero, but by different convergence result
- We can further prove **uniform convergence** over a compact subset of complex plane, if it doesn't contain any zero nor pole of $\bar{g}(s)$
- Convergence of transfer matrix is **related to time-domain response** by Inverse Laplace Transform
- Extensions for random network ensembles $\bar{g}(s) = (E_w[\, g^{-1}(s,w)])^{-1}$

# Effect of Network Algebraic Connectivity

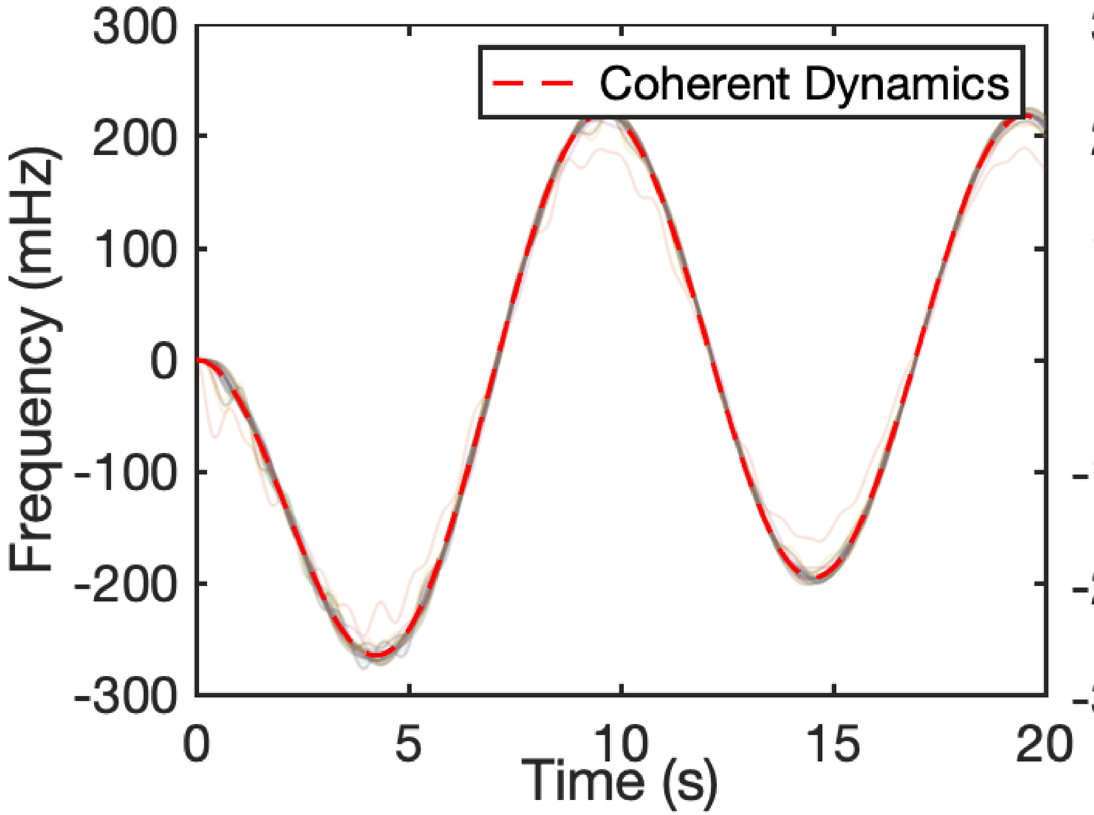
$\lambda_2(L) \uparrow$

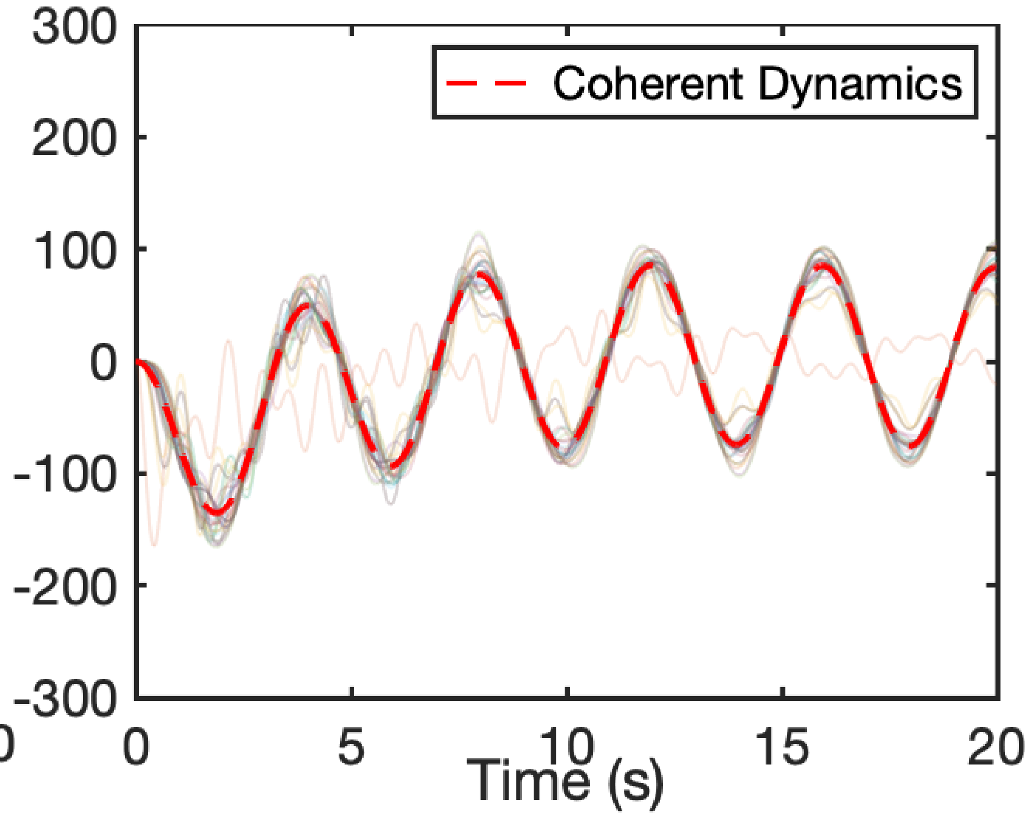



Coherent dynamics acts as a more accurate version of the Center of Inertia (CoI)

# Sinusoidal Disturbances: $\sin(\boldsymbol{\omega_d t})$

$\omega_d \uparrow$



# Outline

- Characterization of Coherent Dynamics [Min, M ‘21]
- Reduced-Order Model of Coherent Response [Min, Paganini, M ‘21]

# Accurate Reduced-Order Models for Heterogeneous Coherent Generators

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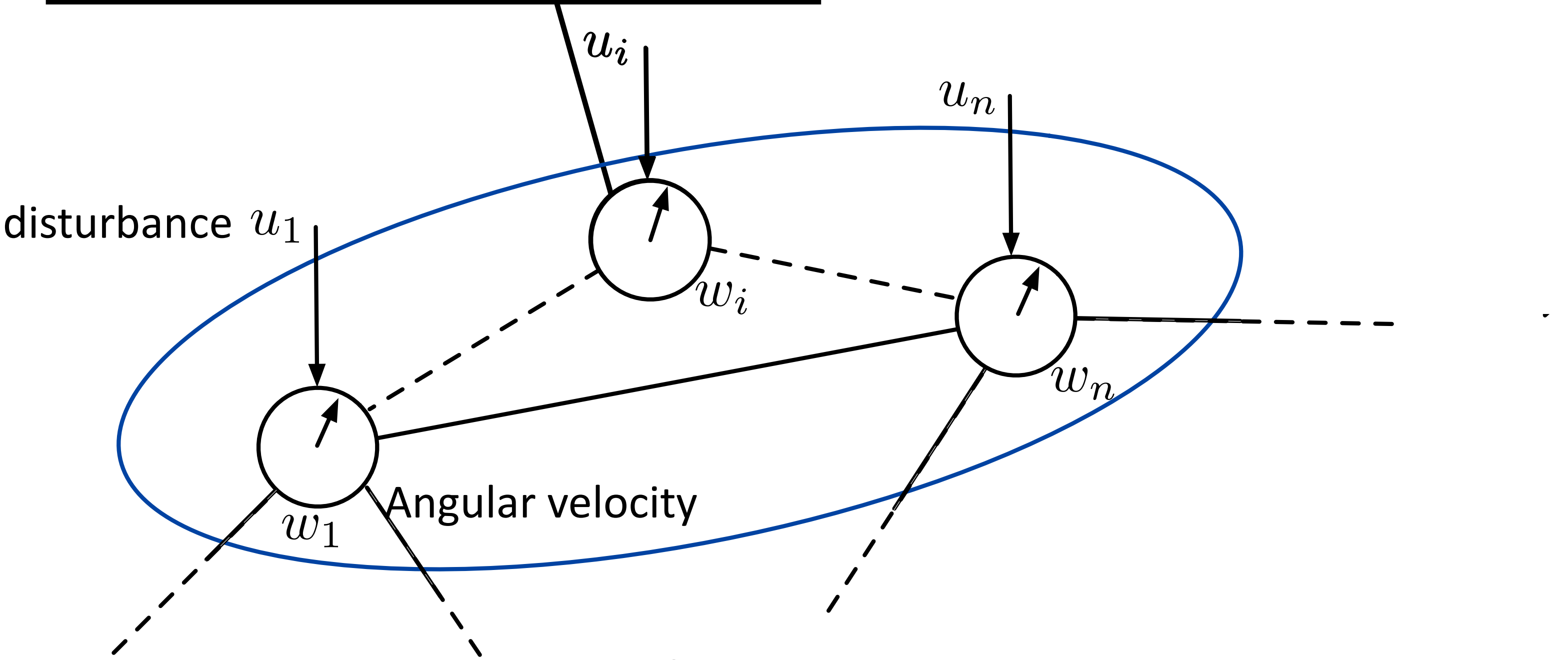
# Aggregation of Coherent Generators

$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}}$$

$m_i$: inertia
$d_i$: damping coefficient
$r_i^{-1}$: droop coefficient
$\tau_i$: turbine time constant

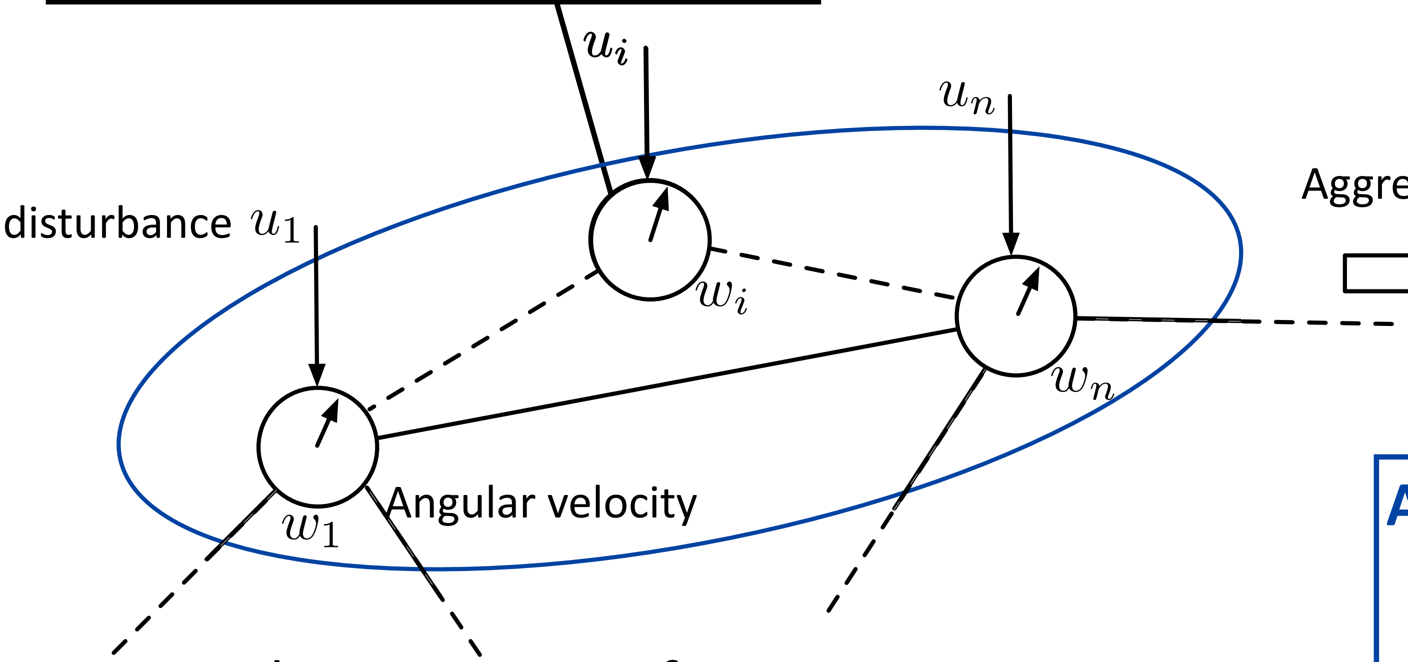



coherent group of n generators

# Aggregation of Coherent Generators

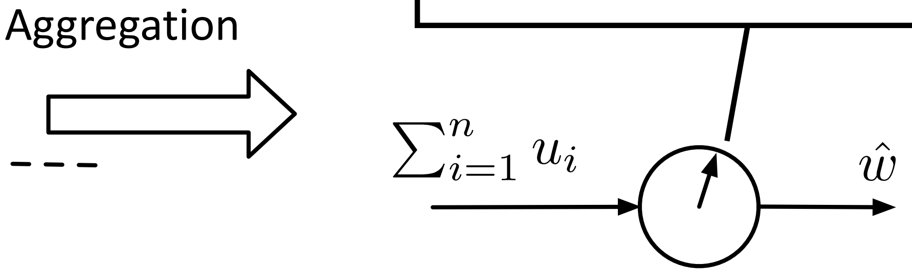
$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}}$$

**Question:** How to choose the different parameters of $\hat{g}(s)$?




coherent group of n generators

$$\hat{g}(s) = \frac{1}{\hat{m}s + \hat{d} + \frac{\hat{r}^{-1}}{\hat{\tau}s+1}}$$




**Answer:** Use instead

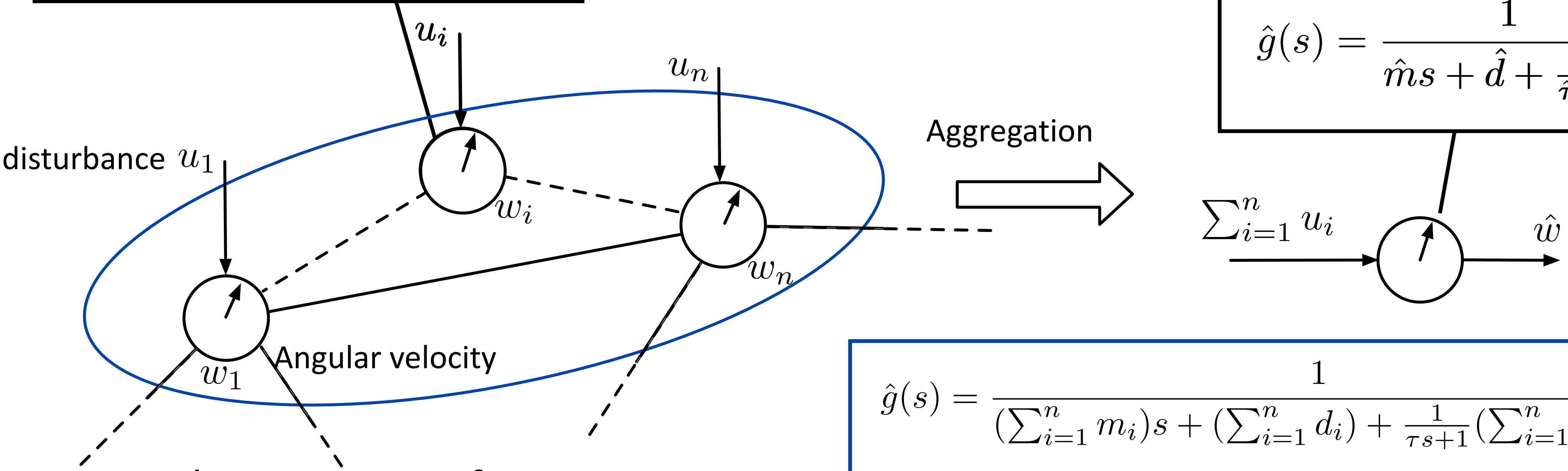
$$\hat{g}(s) = \frac{1}{n}\bar{g}(s) = \left(\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1}$$

# Aggregation for Homogeneous $\tau_i = \tau$

$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s+1}}$$

suppose $\tau_i = \tau$

then $\hat{m} = \sum_{i=1}^{n} m_i, \quad \hat{d} = \sum_{i=1}^{n} d_i, \quad \hat{r}^{-1} = \sum_{i=1}^{n} r_i^{-1}$




$$\hat{g}(s) = \frac{1}{\hat{m}s + \hat{d} + \frac{\hat{r}^{-1}}{\hat{\tau}s+1}}$$

coherent group of n generators

$$\hat{g}(s) = \frac{1}{(\sum_{i=1}^n m_i)s + (\sum_{i=1}^n d_i) + \frac{1}{\tau s+1}(\sum_{i=1}^n r_i^{-1})}$$

# Challenges on Aggregating Coherent Generators

For generator dynamics given by a swing model with turbine control:

$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}}$$

The aggregate dynamics:

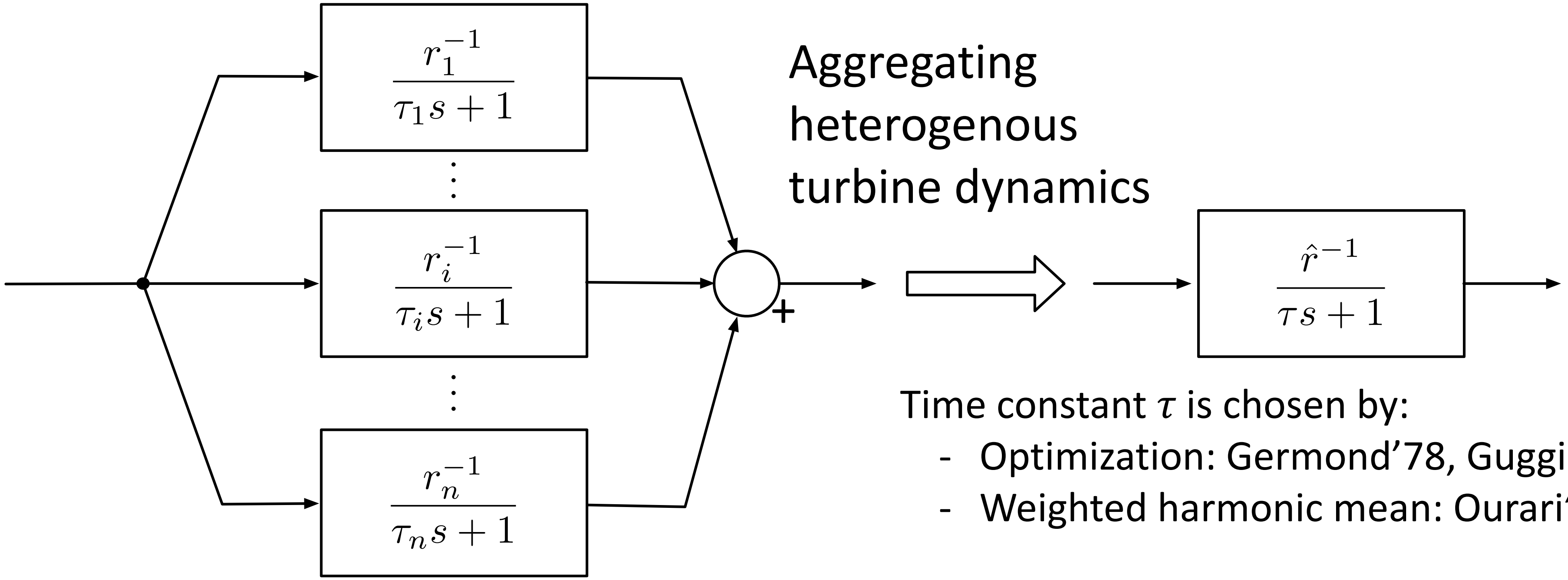
$$\hat{g}(s) = \frac{1}{\hat{m}s + \hat{d} + \boxed{\sum_{i=1}^{n} \frac{r_i^{-1}}{\tau_i s + 1}}}$$

high-order if $\tau_i$ are heterogeneous

Need to find a low-order approximation of $\hat{g}(s)$

# Prior Work: Aggregation for heterogeneous $\tau_i$s

When time constants are **heterogenous**:




Time constant $\tau$ is chosen by:
- Optimization: Germond'78, Guggilam'18
- Weighted harmonic mean: Ourari'06

**Drawbacks:**
- the order of overall approximation model is restricted to 2nd order
- the only "decision variable" is the time constant
- does not consider the effect of inertia or damping in the approx.

Inaccurate Approximation

# Balanced Truncation

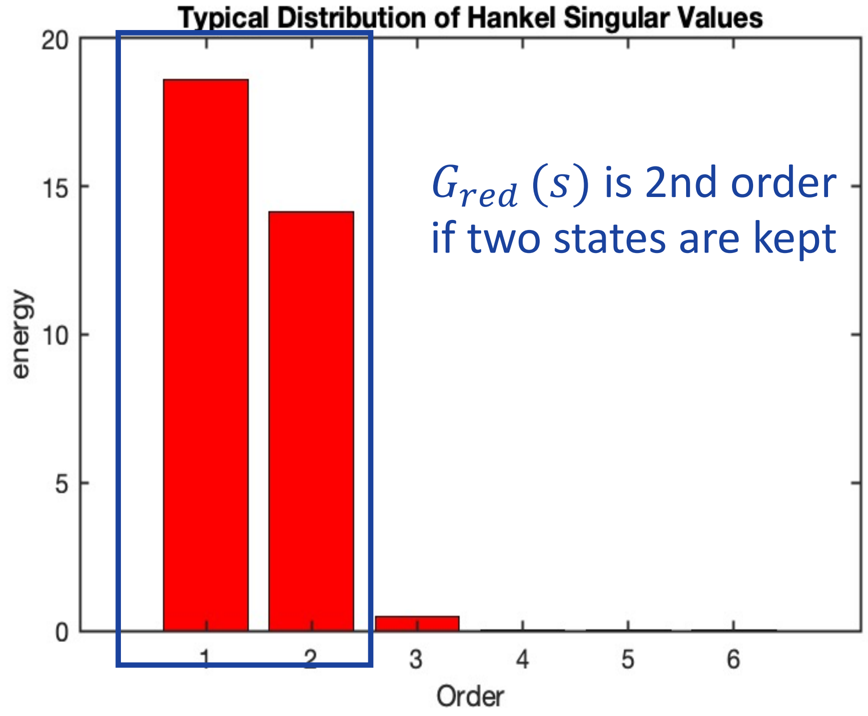
A model reduction method on stable system $G(s)$ such that:

- The reduced model $G_{red}(s)$ is stable
- The error in $H_\infty$-norm:

$$\|G(s) - G_{red}(s)\|_{\mathcal{H}_\infty}$$

is upper bounded by a small value that depends on $G(s)$ and **the order of $G_{red}(s)$**

k-th order $G_{red}(s)$ is obtained by only keeping states of $G(s)$ associated with k largest Hankel Singular Value




**There is DC gain mismatch between $G(s)$ and $G_{red}(s)$!!**

# Frequency Weighted Balanced Truncation

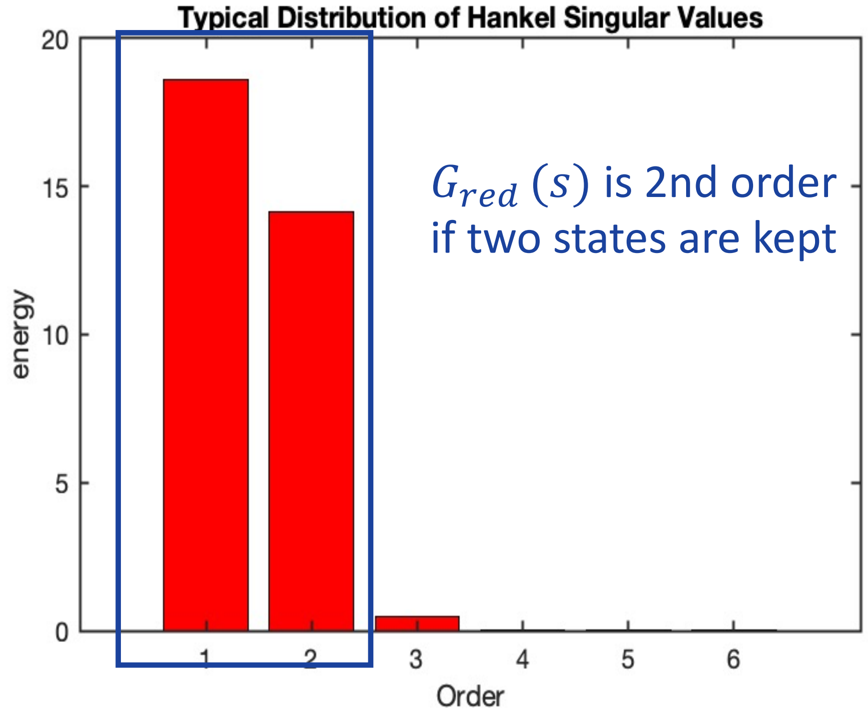
A frequency weighted model reduction method on stable system $G(s)$ such that:

- The reduced model $G_{red}(s)$ is stable
- The frequency weighted error in $H_\infty$-norm:

$$||W(s)(G(s) - G_{red}(s))||_{\mathcal{H}_\infty}$$

is upper bounded by a small value that depends on $G(s)$ and **the order of** $G_{red}(s)$ ) and $W(s)$

k-th order $G_{red}(s)$ is obtained by only keeping states of $G(s)$ associated with k largest frequency weighted Hankel Singular Value




**The DC gain mismatch between $G(s)$ and $G_{red}(s)$ can be made arbitrarily small weighting higher low freqs.**

# Aggregation Model by Frequency Weighted Balanced Truncation

Two approaches to get a k-th order reduction model of aggregate dynamics $\hat{g}(s)$:

- (k-1)-th order balanced truncation on high-order turbine dynamics

$$\tilde{g}_k^{tb}(s) = \frac{1}{\hat{m}s + \hat{d} + \boxed{\tilde{g}_{t,k-1}(s)}}$$

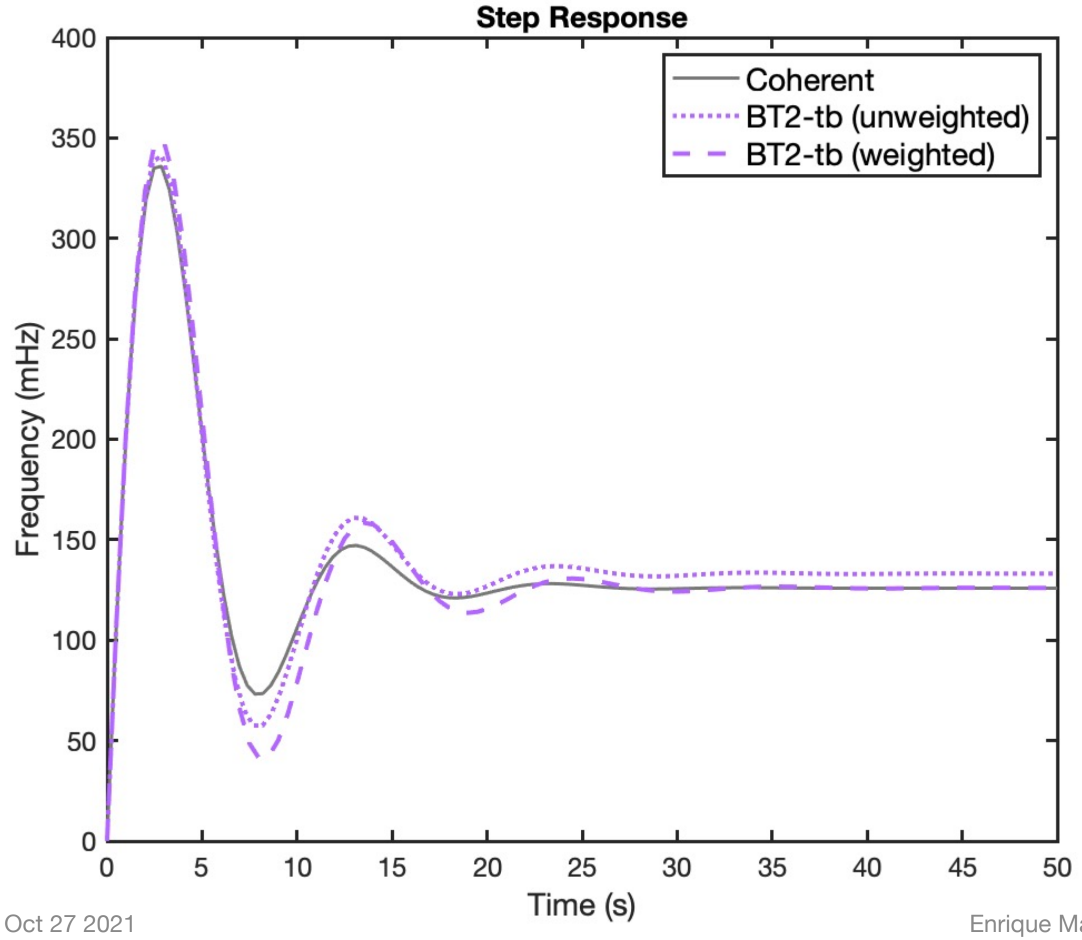
(k-1)-th reduction model on $\sum_{i=1}^{n} \frac{r_i^{-1}}{\tau_i s + 1}$

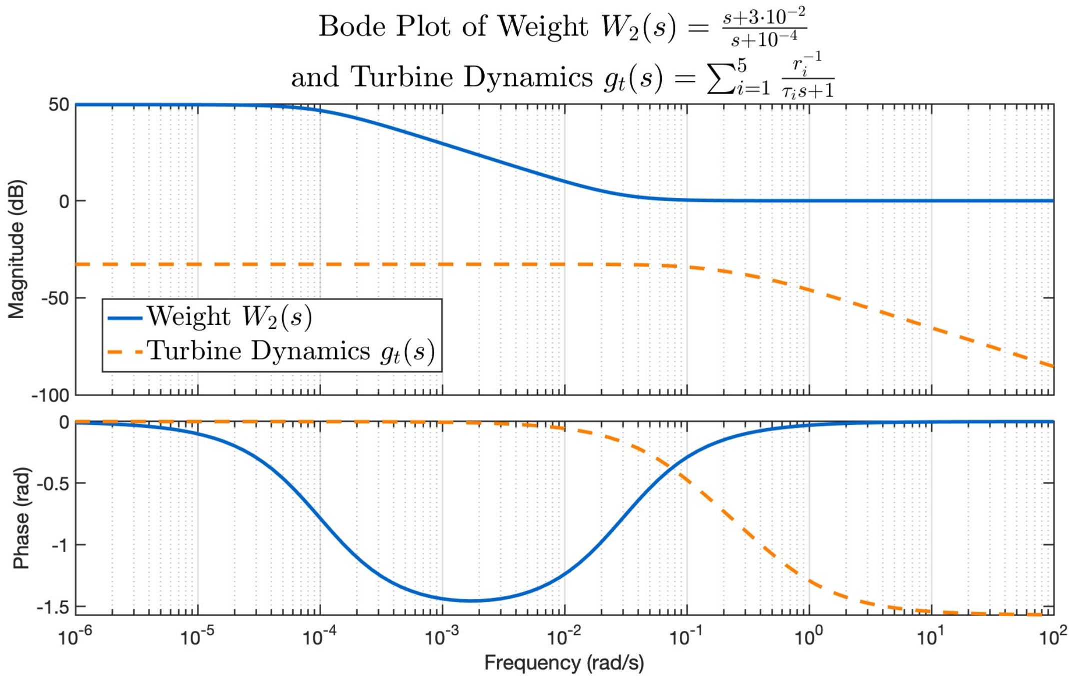
- k-th order balanced truncation on closed-loop dynamics $\hat{g}(s)$

# Numerical Simulation—Matching DC Gain in Balanced Truncation

Compare 2nd order model by balanced truncation on turbine dynamics

with different weights: $W_1(s) = 1$ (unweighted) $W_2(s) = \dfrac{s + 3 \cdot 10^{-2}}{s + 10^{-4}}$ (weighted)



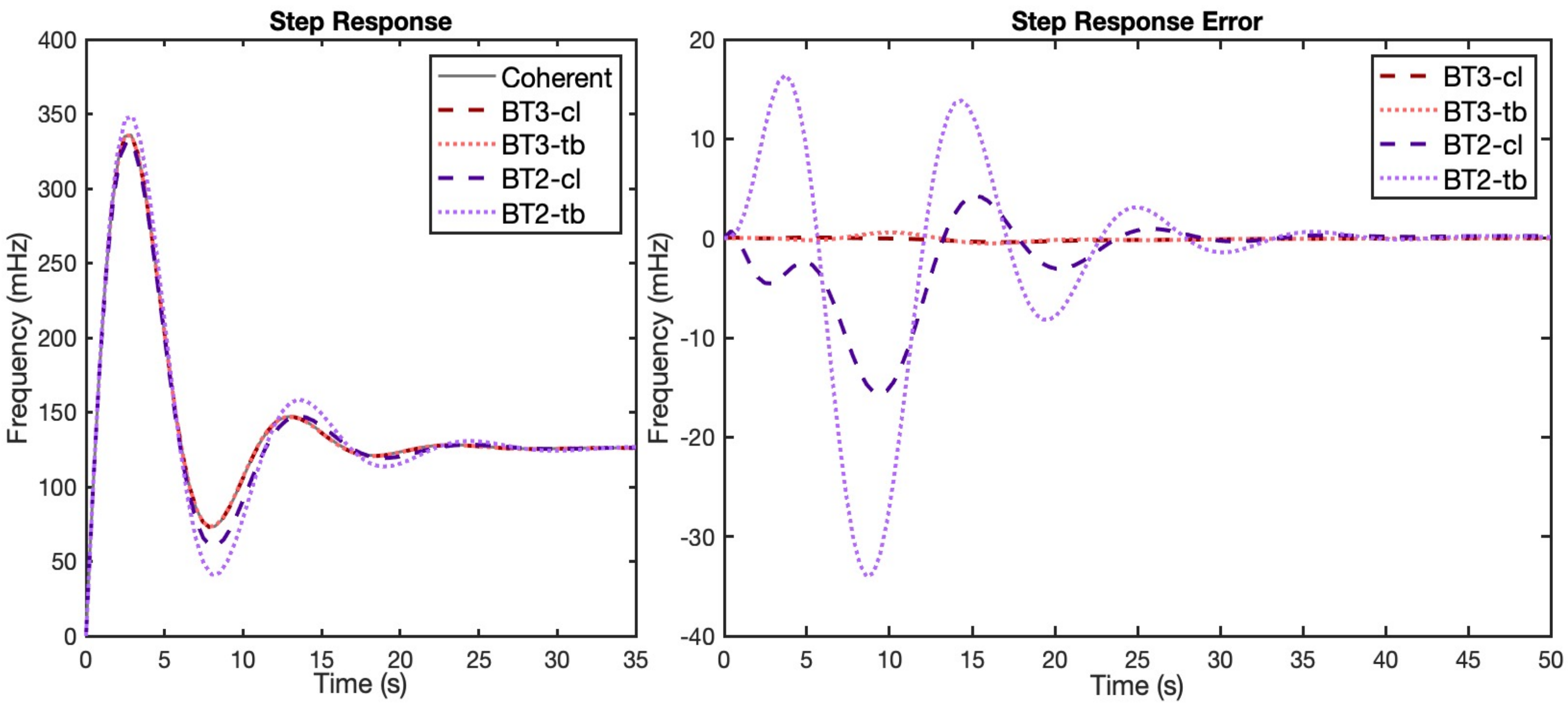




DC gain is matched by putting more weights on low frequency range

# Numerical Simulation—Compare Models by Balanced Truncation

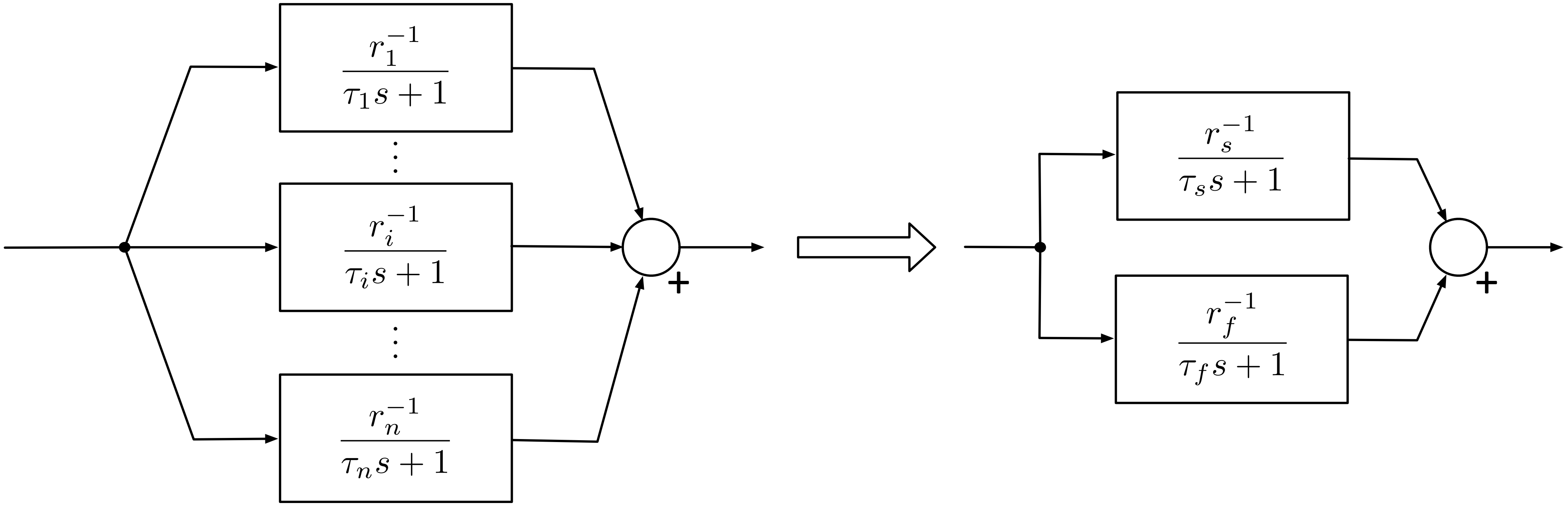
We compare the following 4 reduced order models:

- Balanced truncation on **turbine** dynamics with weight $W_{tb}(s) = \frac{s+3\cdot10^{-2}}{s+10^{-4}}$
  - 2nd order (BT2-tb)
  - 3rd order (BT3-tb)
- Balanced truncation on **closed-loop** dynamics with weight $W_{cl}(s) = \frac{s+8\cdot10^{-2}}{s+10^{-4}}$
  - 2nd order (BT2-cl)
  - 3rd order (BT3-cl)




- 3rd order models are almost accurate
- balanced truncation on closed-loop is better than on turbine dynamics, given the same order

# Interpretation of 3rd Order Reduced Model




- The high-order turbine dynamics can be **almost accurately** recovered by **two turbines** in parallel
- Such approximation works for aggregating even more turbines than in the test case

# Summary

- Frequency domain characterization of **coherent dynamics**, as a low rank property of the transfer function.

- **Coherence is** a **frequency dependent** property:
  - Effective algebraic connectivity $f(s)\lambda_2(L)$
  - Disturbance frequency spectrum

- We use frequency **weighted balanced truncation** to suggest possible improvements to obtain accurate reduced order model of aggregated dynamics of coherent generators:

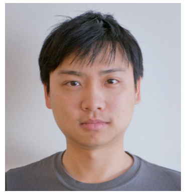
  - increase model complexity ($3^{rd}$ order/two turbines)

  - model reduction on closed-loop dynamics

# Thanks!

**Related Publications:**

- Min, M, “Coherence and Concentration in Tightly Connected Networks,” **submitted**
- Min, Paganini, M, “Accurate Reduced Order Models for Coherent Synchronous Generators,” **L-CSS 2021**
- Jiang, Bernstein, Vorobev, M, “Grid-forming Frequency Shaping Control,” **L-CSS 2021**



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