Reconstructing Networks from Partial Measurements

Philippe Jacquod

CCS2021 Satellite Symposium

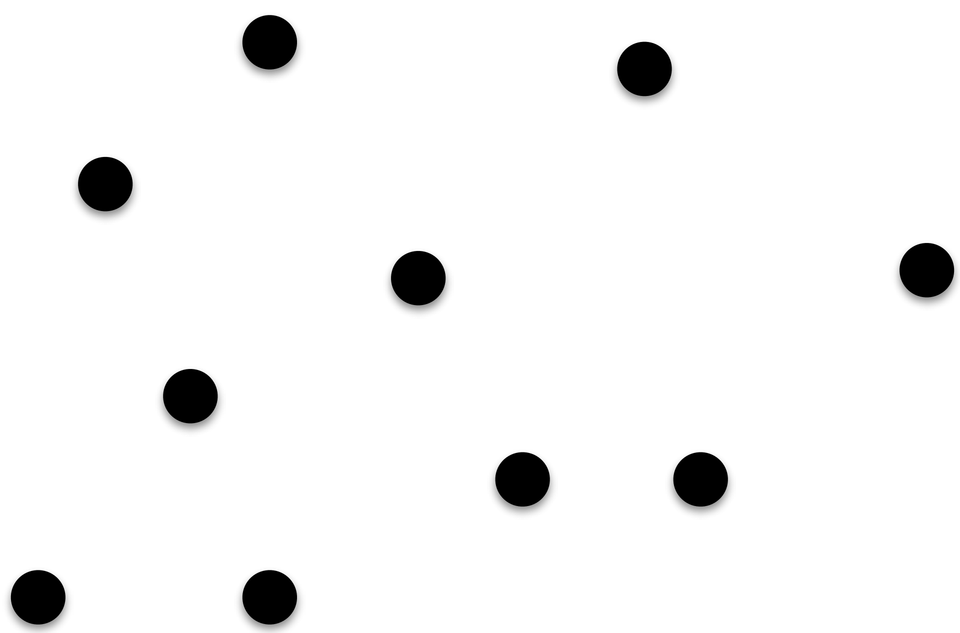
Colls.: R. Delabays (UCSB)
M. Tyloo (Uni GE - LANL)

M Tyloo, R Delabays, and PJ, Chaos 31, 103117 (2021)

The problem

* agents
* their degrees of freedom

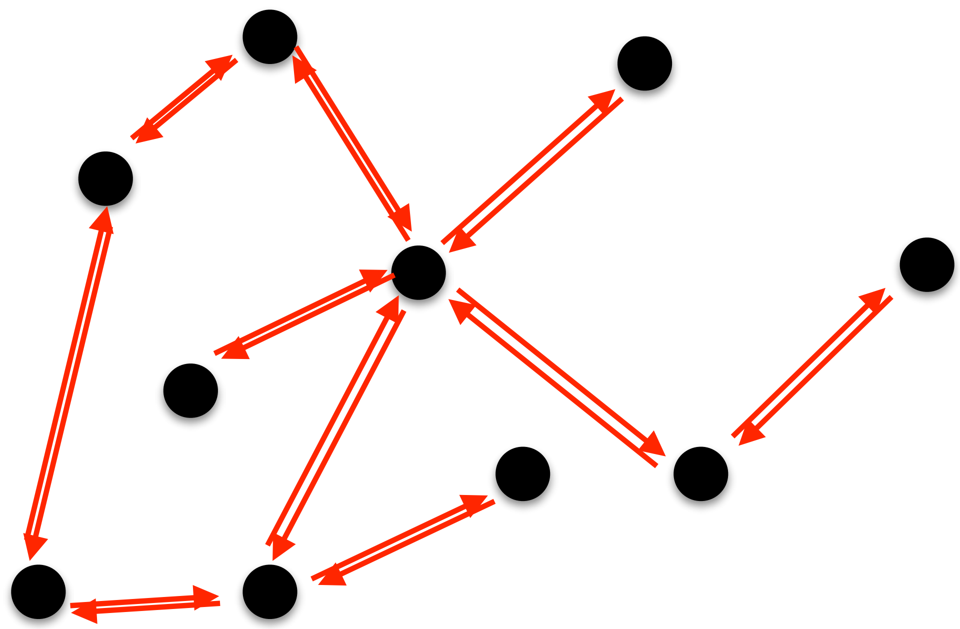
$$\{x_i(t), \dot{x}_i(t)\}$$



The problem

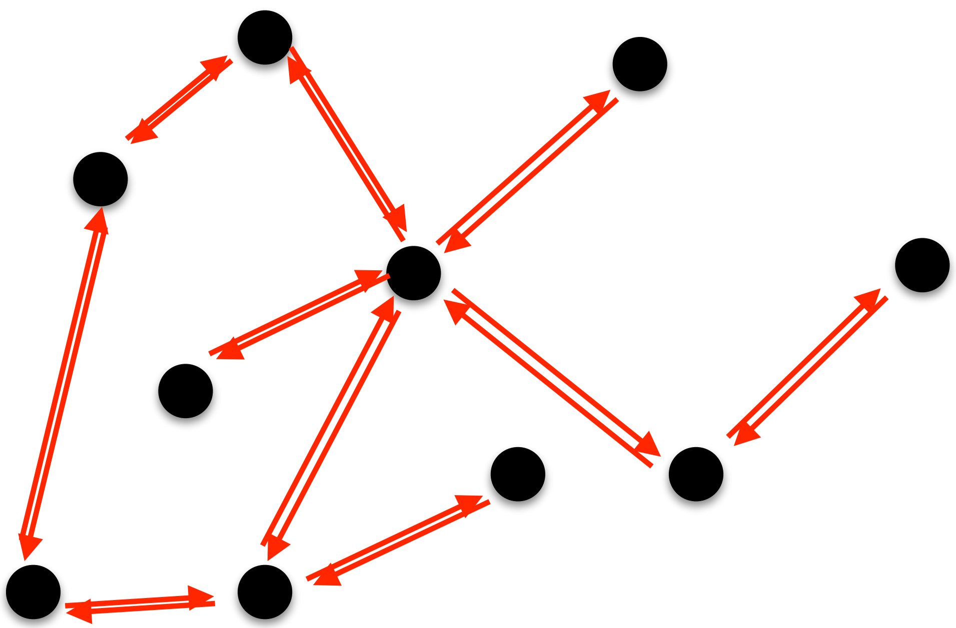
* n agents
* their degrees of freedom
* what can we know of the way they interact ?

$$\{x_i(t), \dot{x}_i(t)\}$$



The problem

$\{x_i(t), \dot{x}_i(t)\}$



What we want to extract :

* Number n of agents ?
* Connectivity ? Graph topology ?

From
-complete / partial
-active / passive
measurements.

Previous works

* Probing, i.e. injecting controlled signal and measuring the response

D. Yu, M. Righero, and L. Kocarev, PRL 2006
M. Timme, PRL 2007
D. Yu and U. Parlitz, EPL 2008
F. Basiri, J. Casadiego, M. Timme, and D. Witthaut, PRE 2018
M. Tyloo and R. Delabays, J Phys Complex 2021

* Optimization of likelihood cost function

D.-T. Hoang, J. Jo, and V. Periwal, PRE 2019
V.A.Makarov, F. Panetsos, and O. de Febo, J. Neurosci. Methods 2005
S.G. Shandilya and M. Timme, NJP 2011
M. J. Panaggio, M.-V. Ciocanel, L. Lazarus, C. M. Topaz, and B. Xu, Chaos 2019

* Short-time dynamics / trajectory correlations

R. Dahlhaus, M. Eichler, and J. Sandkühler, J. Neurosci. Methods 1997
K. Sameshima and L. A. Baccalá, J. Neurosci. Methods 1999
M.E.J. Newman, Nat. Phys. 2018
T. P. Peixoto, PRL 2019
M. G. Leguia, C. G. B. Martínez, I. Malvestio, A. T. Campo, R. Rocamora,
Z. Levnaji´c, and R. G. Andrzejak, PRE 2019
A. Banerjee, J. Pathak, R. Roy, J. G. Restrepo, and E. Ott, Chaos 2019

Noise vs. frequency correlators vs. connectivity

* Two-point correlators

$$C_{ij} = \langle x_i(t) x_j(t) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_0^T x_i(t) x_j(t)\, \mathrm{d}t$$

J. Ren, W.-X. Wang, B. Li, and Y.-C. Lai, PRL 2010
W.-X. Wang, J. Ren, Y.-C. Lai, and B. Li, Chaos 2012
E. S. C. Ching and H. C. Tam, PRE 2017
Y. Chen, S. Wang, Z. Zheng, Z. Zhang, and G. Hu, EPL 2016
H. C. Tam, E. S. C. Ching, and P.-Y. Lai, Physica A 2018

$$\boldsymbol{C} \propto \boldsymbol{L}^{\dagger}$$

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Reconstruction of the network Laplacian matrix via inversion of the equal time, 2-point correlation matrix

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Reconstruction of the network Laplacian matrix via inversion of the equal time, 2-point correlation matrix

- Either you have the full matrix, i.e. from a complete measurement, or you have nothing.

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Reconstruction of the network Laplacian matrix via inversion of the equal time, 2-point correlation matrix

- Either you have the full matrix, i.e. from a complete measurement, or you have nothing.
- What can we do if we access only to a subset of all agents ?

Earlier works

PHYSICAL REVIEW LETTERS **120**, 084101 (2018)

Robustness of Synchrony in Complex Networks and Generalized Kirchhoff Indices

M. Tyloo,[1,2] T. Coletta,[1] and Ph. Jacquod[1]

[1]*School of Engineering, University of Applied Sciences of Western Switzerland HES-SO, CH-1951 Sion, Switzerland*

[2]*Institute of Physics, EPF Lausanne, CH-1015 Lausanne, Switzerland*

Trace of frequency correlation matrix = trace of graph Laplacian

Trace of position correlation matrix = trace of inverse Laplacian

Earlier works

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Trace of frequency correlation matrix = trace of graph Laplacian
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SCIENCE ADVANCES | RESEARCH ARTICLE

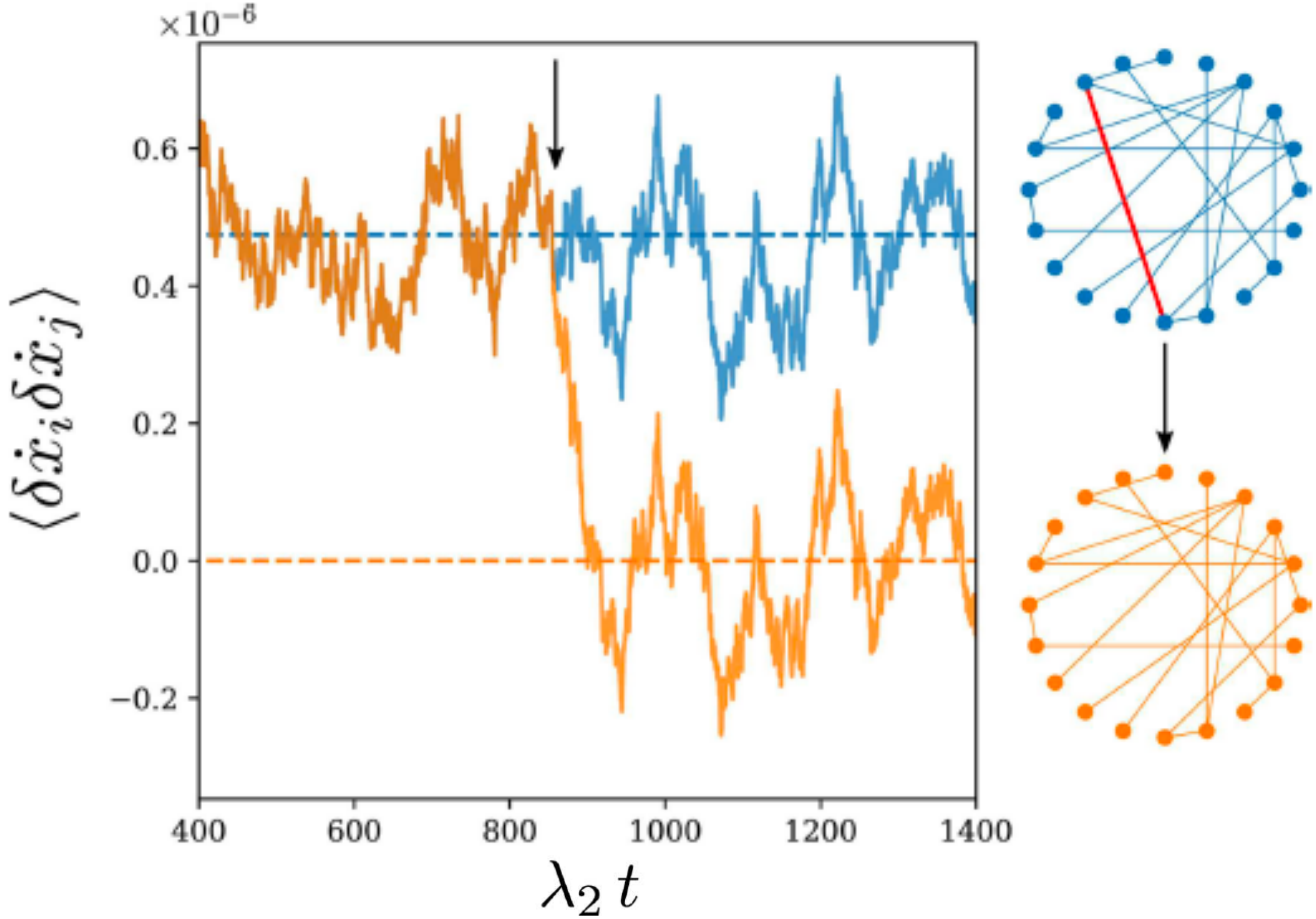
APPLIED SCIENCES AND ENGINEERING

The key player problem in complex oscillator networks and electric power grids: Resistance centralities identify local vulnerabilities

M. Tyloo[1,2], L. Pagnier[1,2], P. Jacquod[2,3]*

Diagonal frequency correlators = diagonal elements of graph Laplacian
Diagonal position correlators = diagonal elements of inverse Laplacian

Noise vs. frequency correlators vs. connectivity



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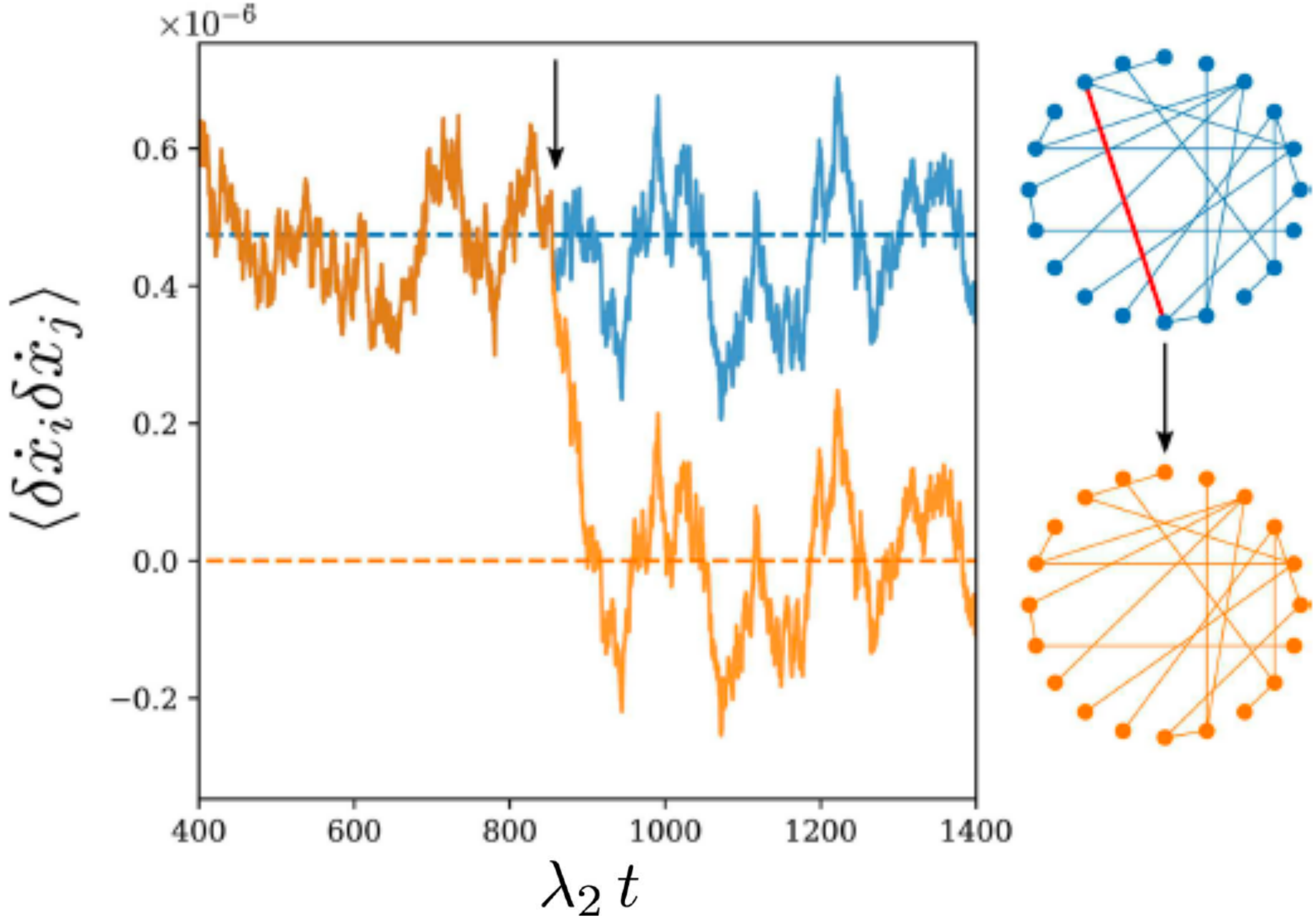
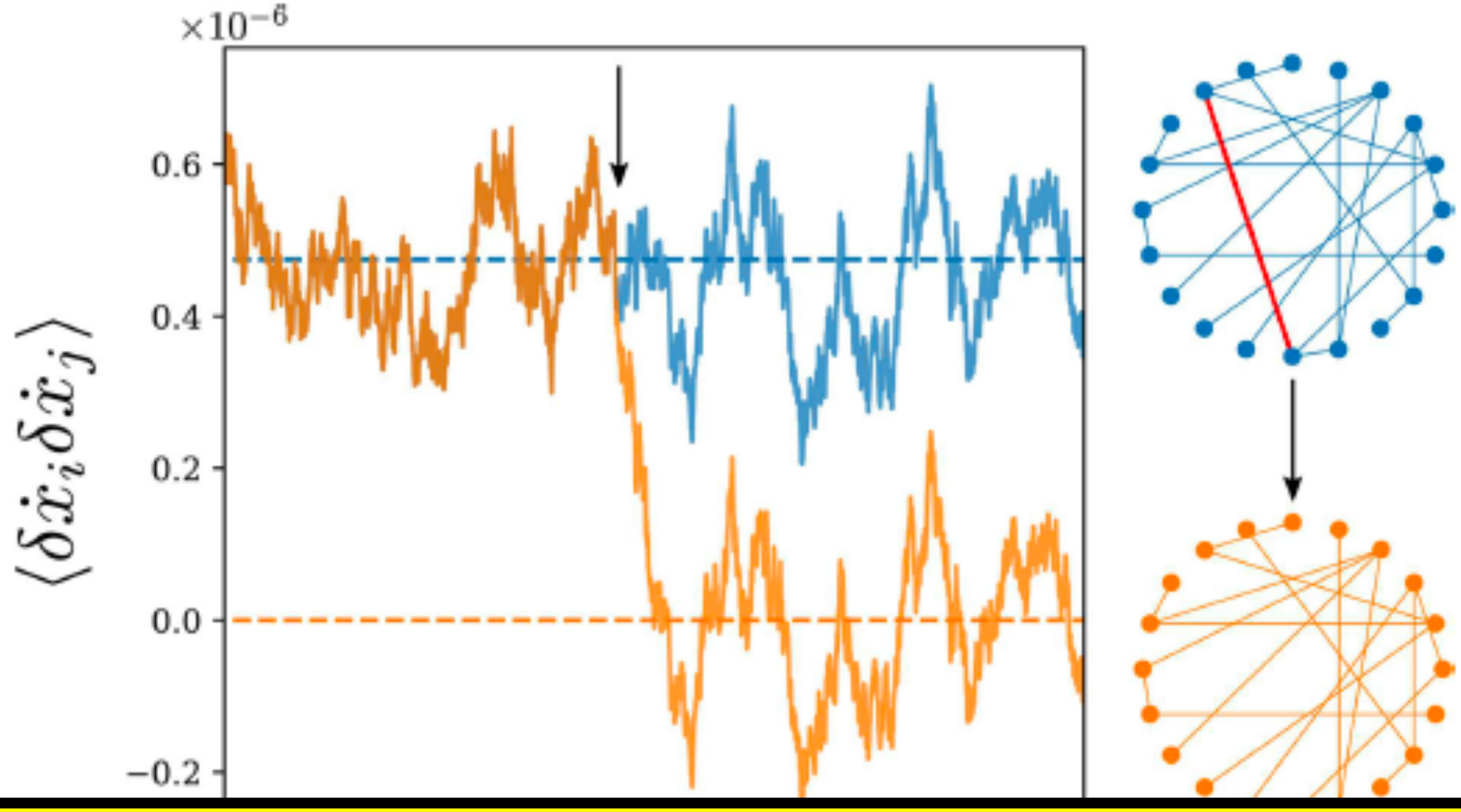


$$\langle \delta \dot{x}_i \delta \dot{x}_j \rangle = \xi_0^2 \sum_{k=q}^{\infty} (-\tau_0)^k \left(\mathbb{J}^k\right)_{ij}$$

Noise vs. frequency correlators vs. connectivity



-direct extraction of Laplacian
-partial inference from partial measurements

$$\langle \delta\dot{x}_i \delta\dot{x}_j \rangle = \xi_0^2 \sum_{k=q}^{\infty} (-\tau_0)^k \left(\mathbb{J}^k\right)_{ij}$$

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Sketch of the analytics (i)

The model

Unperturbed dynamics $\dot{\mathbf{x}}(t) = \mathbf{F}[\mathbf{x}(t)]$ $\mathbf{F}[\mathbf{x}^*] = 0$

Linearization about steady-state + perturbation

$$\boldsymbol{\delta}\dot{\boldsymbol{x}} = -\mathbb{J}(\boldsymbol{x}^*)\,\boldsymbol{\delta}\boldsymbol{x} + \boldsymbol{\xi}$$

Network/coupling structure

$$\mathbb{J}_{ij}(\boldsymbol{x}^*) = -\partial F_i(\boldsymbol{x}^*)/\partial x_j$$

Assumptions

- $\mathbb{J}(\boldsymbol{x}^*)$ is symmetric and positive semidefinite (undirected coupling; stable fixed point)

Sketch of the analytics (ii)

Modal decomposition of J

Real eigenvalues $0 \leq \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$

Orthogonal eigenbasis $\{\boldsymbol{u}_\alpha\}_{\alpha=1}^{n}$

$$\boldsymbol{\delta x}(t) = \sum_\alpha c_\alpha(t)\boldsymbol{u}_\alpha$$

- Langevin equation for expansion coefficients

$$\dot{c}_\alpha(t) = -\lambda_\alpha c_\alpha(t) + \mathbf{u}_\alpha \cdot \boldsymbol{\xi}(t)$$

- Solutions

$$c_\alpha(t) = e^{-\lambda_\alpha t} \int_0^t e^{\lambda_\alpha t} \mathbf{u}_\alpha \cdot \boldsymbol{\xi}(t')\mathrm{d}t'$$

- Velocity correlator

$$\langle \delta\dot{x}_i(t)\delta\dot{x}_j(t)\rangle = \sum_{\alpha,\beta} \langle \dot{c}_\alpha(t)\dot{c}_\beta(t)\rangle u_{\alpha,i}u_{\beta,j}$$

Sketch of the analytics (iii)

Two-point velocity correlators

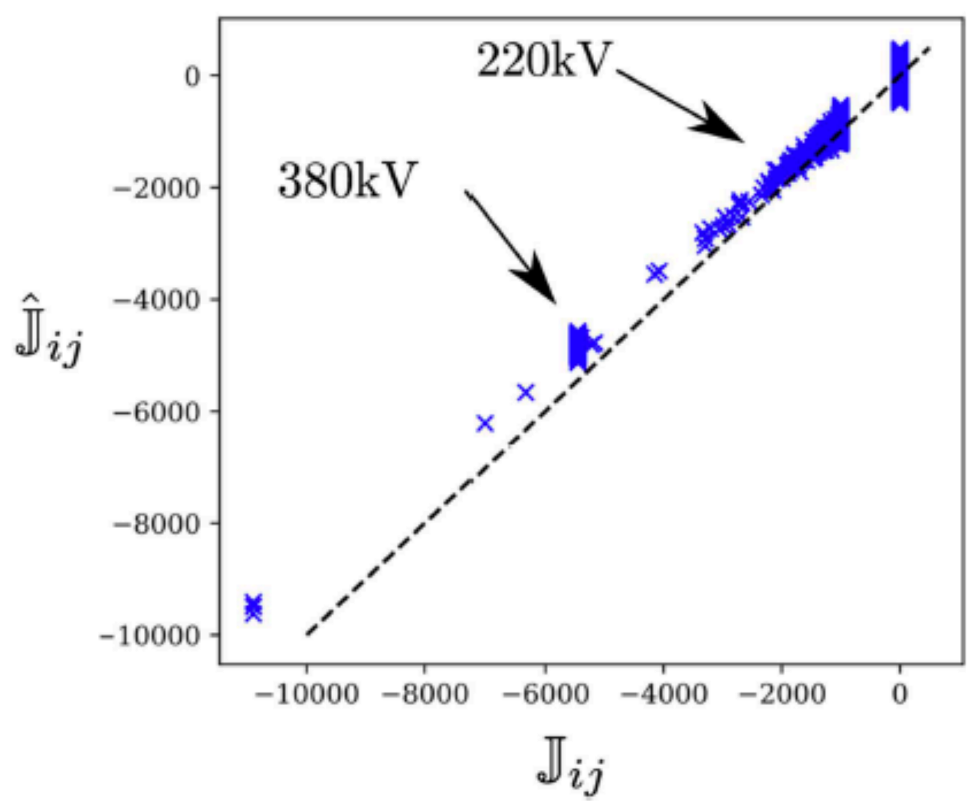
Need to define first and second moment of noise
-> Orstein-Uhlenbeck

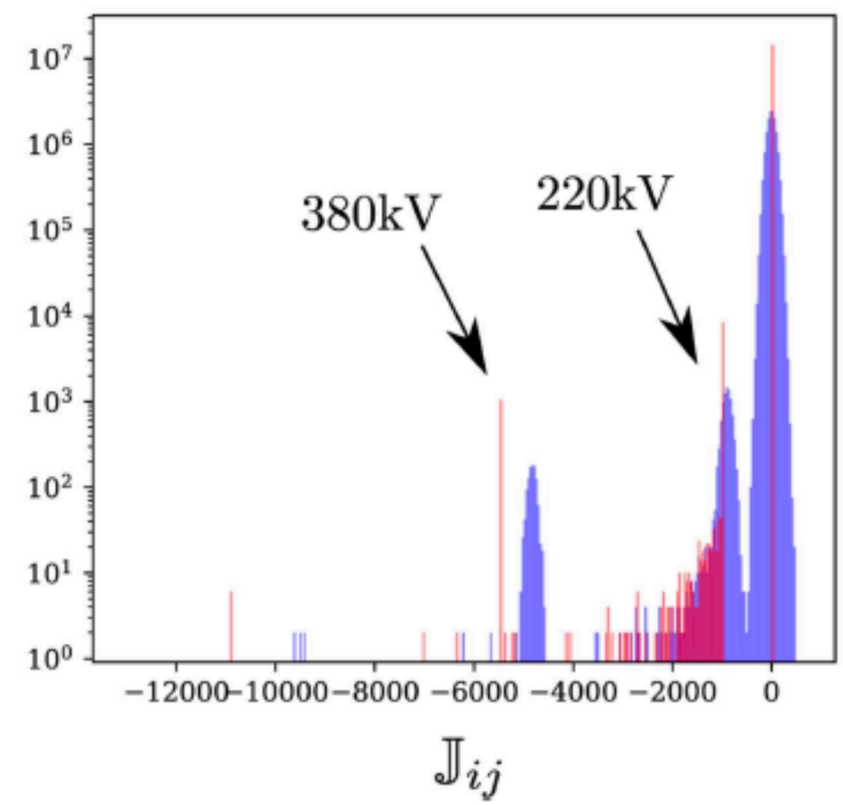
$$\langle \xi_i(t) \rangle = 0 \qquad \langle \xi_i(t + \Delta t/2) \xi_j(t - \Delta t/2) \rangle = \xi_0^2 \, \delta_{ij} \exp\left(-|\Delta t|/\tau_0\right)$$

$$\lim_{t\to\infty} \langle \delta \dot{x}_i(t) \delta \dot{x}_j(t) \rangle = \xi_0^2 \left[\delta_{ij} + \sum_{k=1}^{\infty} (-\tau_0)^k (\mathbb{J}^k)_{ij} \right]$$

Note : $\langle \cdots \rangle = \lim_{T\to\infty} T^{-1} \int_0^T \cdots \, \mathrm{d}t$

Direct reconstruction



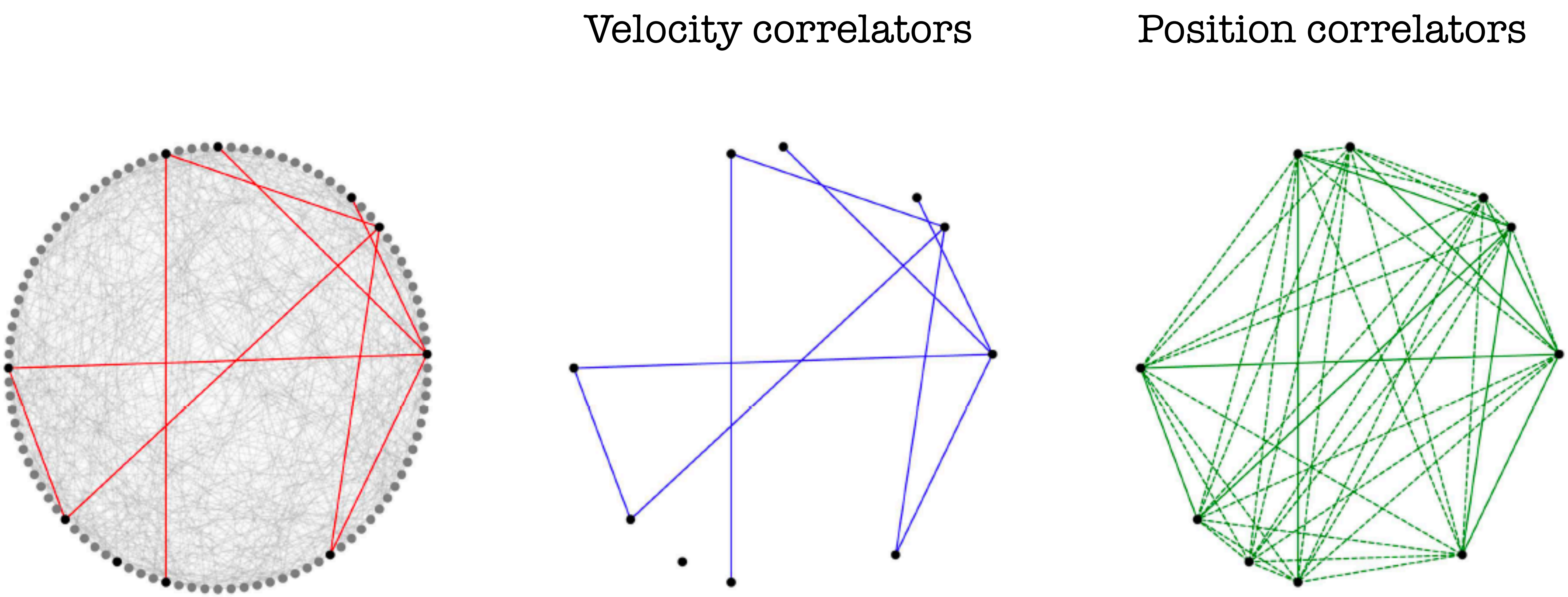


Relatively short correlation time

$$\hat{\mathbb{J}}_{ij} = (\delta_{ij} - \langle \delta\dot{x}_i \delta\dot{x}_j \rangle / \xi_0^2)\, \tau_0^{-1}$$

L. Pagnier and P. Jacquod, PanTaGruEl, Zenodo Repository (2019). doi.org/10.5281/zenodo.2642175

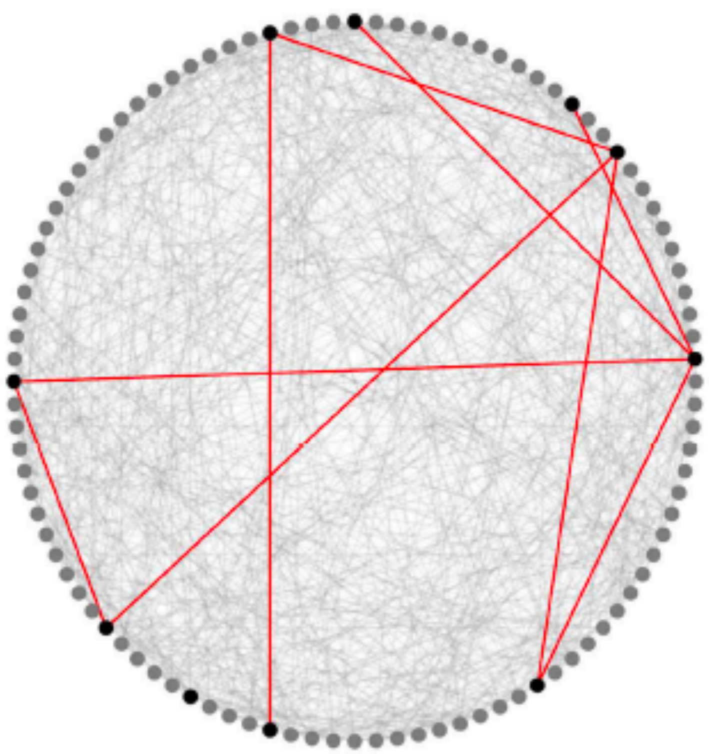
Partial reconstruction (i) : n=100 Erdös-Rényi

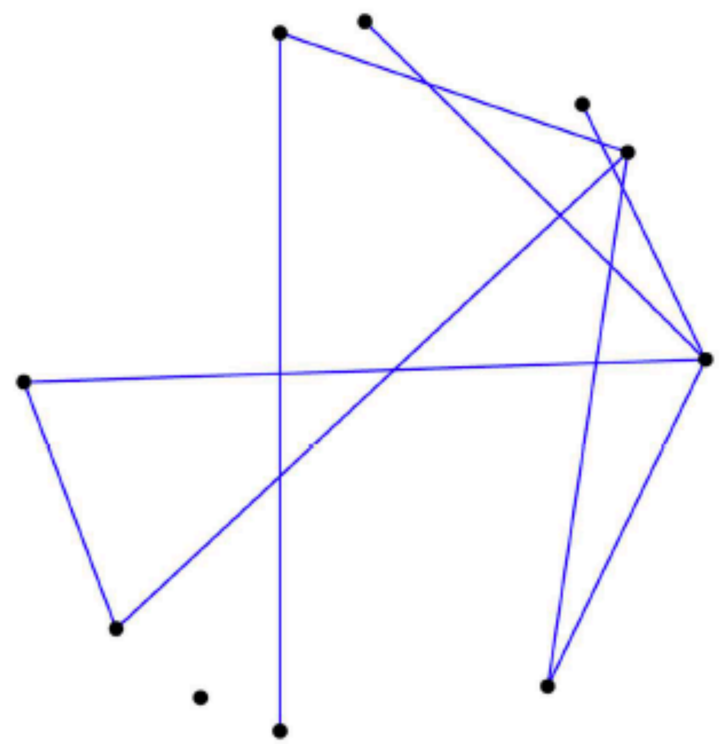


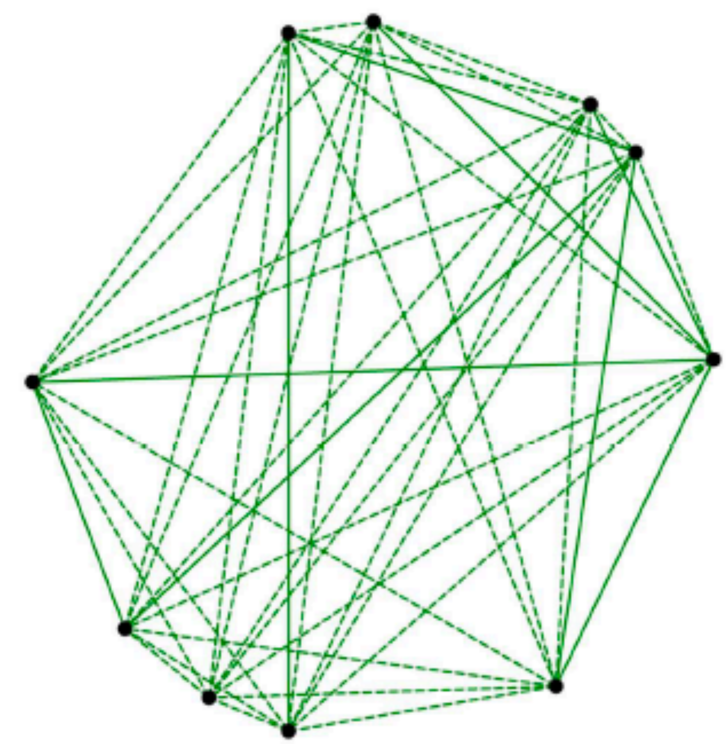
Partial reconstruction (i) : n=100 Erdös-Rényi

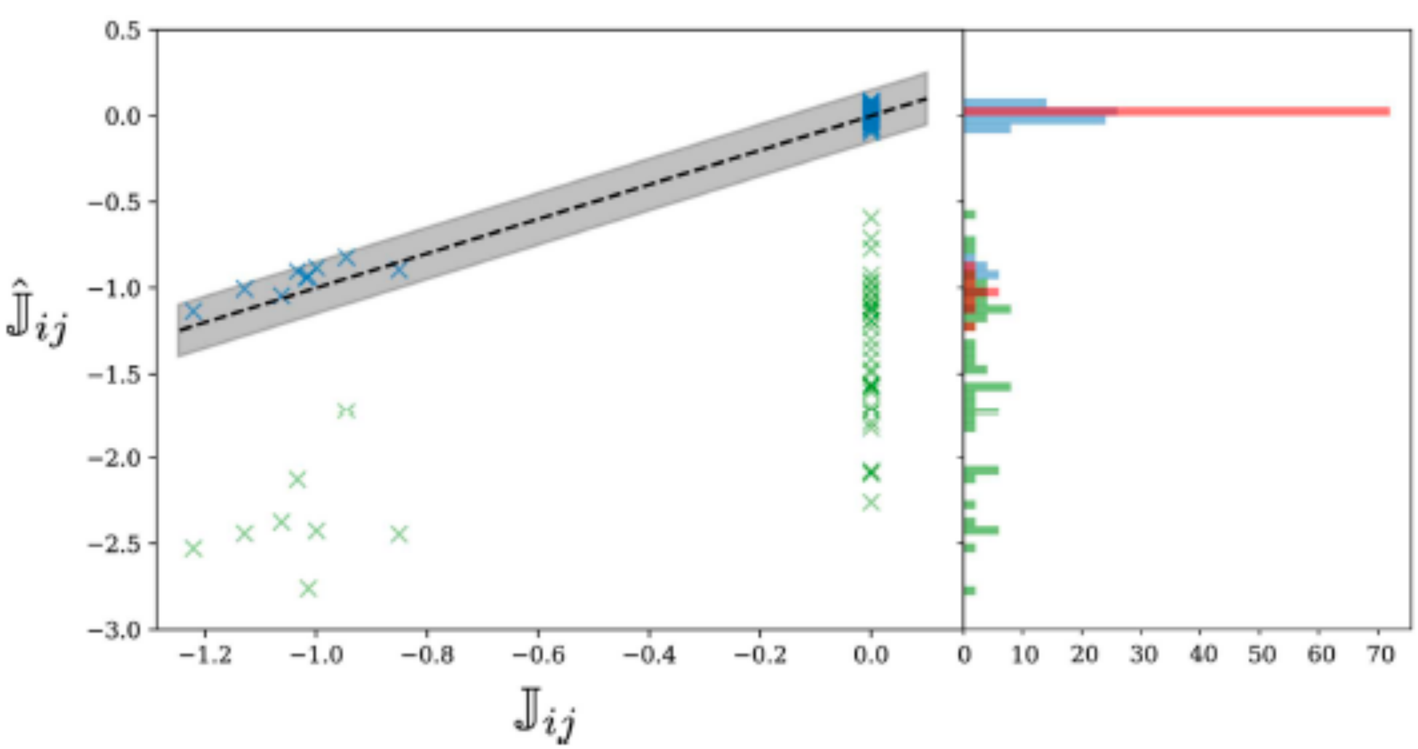
Velocity correlators

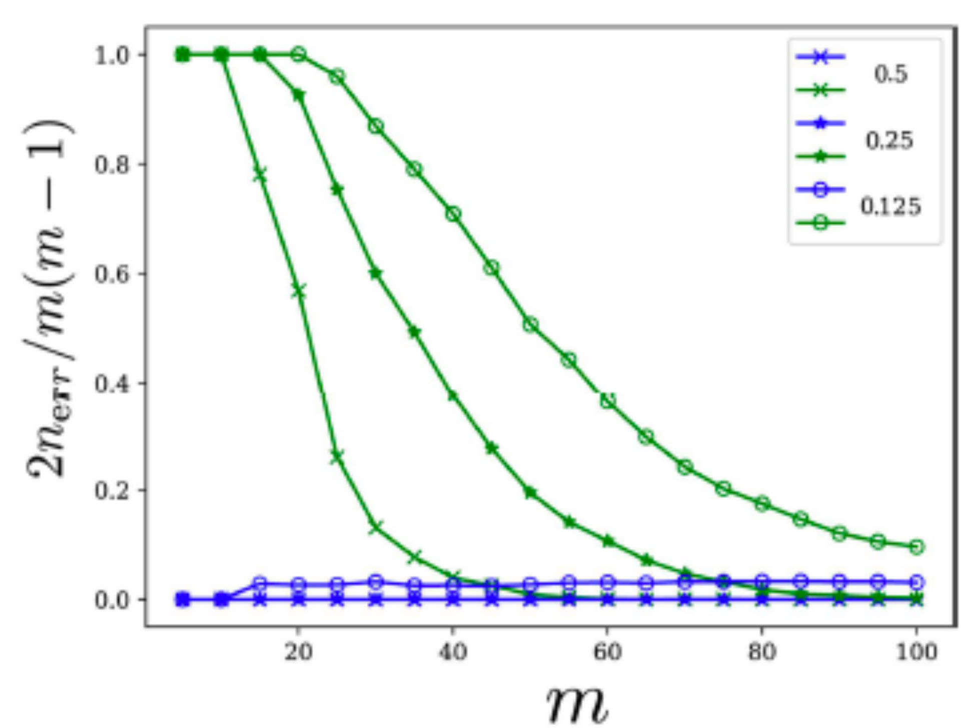
Position correlators



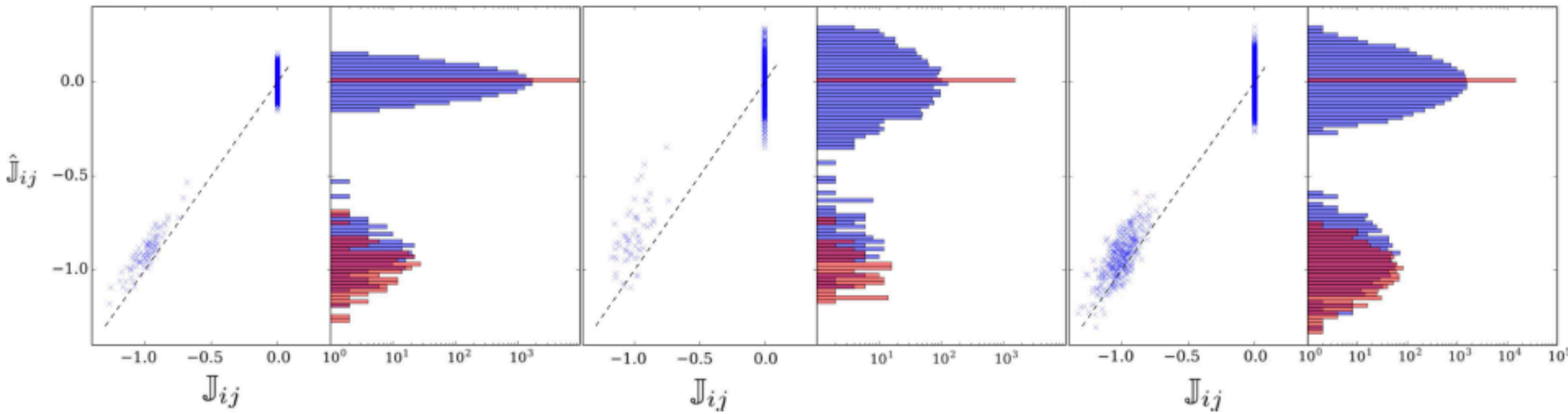






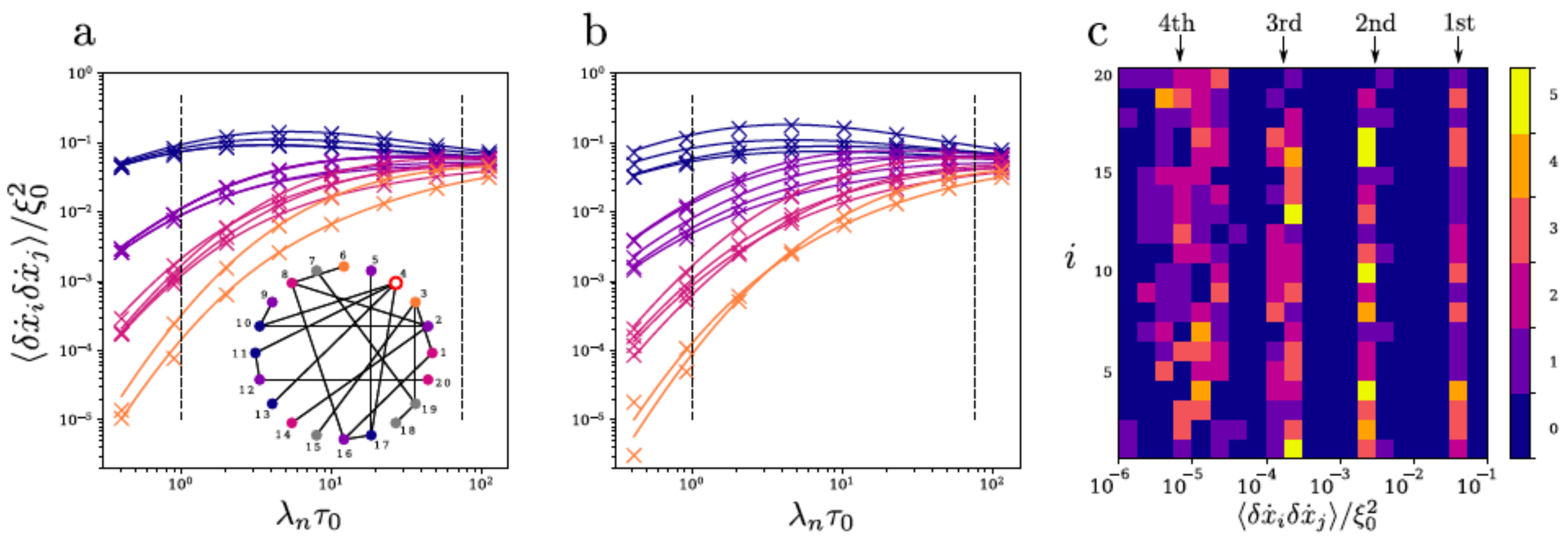


Partial reconstruction (ii) : n=1000 with m=100 observable



Erdös-Rényi Barabasi-Albert Watts-Strogatz

Geodesic distance



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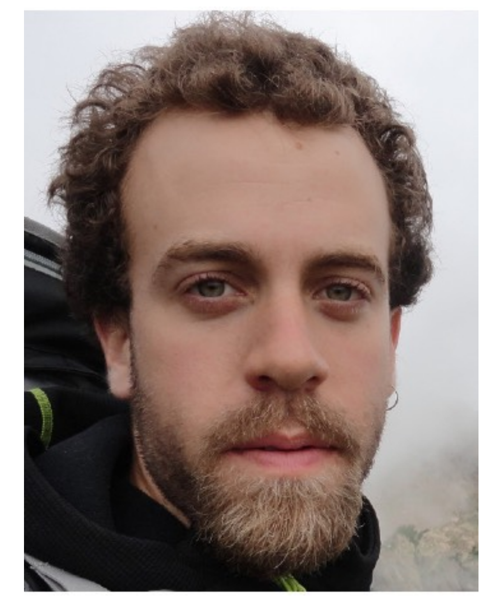
Thank U's

Laurent Pagnier
U of Arizona

Melvyn Tyloo
on the move from GVA to LANL

Tommaso Coletta
Sophia Genetics



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